Initial Semantics for Strengthened Signatures

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We give a new general definition of arity, yielding the companion notions of signature and associated syntax. This setting is modular in the sense requested by [12]: merging two extensions of syntax corresponds to building an amalgamated sum. These signatures are too general in the sense that we are not able to prove the existence of an associated syntax in this general context. So we have to select arities and signatures for which there exists the desired initial monad. For this, we follow a track opened by Matthes and Uustalu [16]: we introduce a notion of strengthened arity and prove that the corresponding signatures have initial semantics (i.e. associated syntax). Our strengthened arities admit colimits, which allows the treatment of the λ-calculus with explicit substitution in the spirit of [12].

1 Introduction

Many programming or logical languages allow constructions which bind variables and this higher-order feature causes much trouble in the formulation, the understanding and the formalization of the theory of these languages. For instance, there is no universally accepted discipline for such formalizations: that is precisely why the POPLmark Challenge [4] offers benchmarks for testing old and new approaches. Although this problem may ultimately concern typed languages and their operational semantics, it already concerns untyped languages. In this work, we extend to new kinds of constructions our treatment of higher-order abstract syntax [13], based on modules and linearity.

First of all, we give a new general definition of arity, yielding the companion notion of signature. The notion is coined in such a way to induce a companion notion of representation of an arity (or of a signature) in a monad: such a representation is a morphism among modules over the given monad, so that an arity simply assigns two modules to each monad. There is a natural category of such representations of a signature and whenever it exists, the initial representation deserves the name of syntax associated with the given signature. This approach enjoys modularity in the sense introduced by [12]: in our category of representations, merging two extensions of a syntax corresponds to building an amalgamated sum.

Our notion of arity (or signature) is too general in the sense that we are not able to build, for each signature, a corresponding initial representation. Following a track opened in Matthes-Uustalu [16], we define a fairly general notion of *strengthened* arity, yielding the corresponding notion of strengthened signature. Our main result (Theorem 7.8) says that any strengthened signature yields the desired initial representation. As usual, this initial object is built as a minimal fixpoint.

Understanding the syntax of the lambda-calculus with explicit substitution was already done in [12], where the arity for this construction was identified as a coend, hence a colimit, of elementary arities (see Section 8). Our main motivation for the present work (and for our next one) was to propose a general approach to syntax (and ultimately to semantics) accounting for this example in the spirit of our previous work [14]. This is achieved thanks to our second main result (Theorem 4.3) which states the existence of colimits in the category of (strengthened) arities.

D. Miller and Z. Ésik (Eds.): Fixed Points
in Computer Science 2012 (FICS 2012)
EPTCS 77, 2012, pp. 31–38, doi:10.4204/EPTCS.77.5

In this extended abstract, we do not discuss proofs. A complete version is available on-line.[1]

2 Related and future work

The idea that the notion of monad is suited for modeling substitution concerning syntax (and semantics) has been retained by many recent contributions on the subject (see e.g. [5, 12, 16]) although some other settings have been considered. For instance in [15] the authors argue in favor of a setting based on Lawvere theories, while in [7] the authors work within a setting roughly based on operads (although they do not write this word down). The latter approach has been broadly extended, notably by M. Fiore [8, 9, 10]. Our main specificity here is the systematic use of the observation that the natural transformations we deal with are linear with respect to natural structures of module (a form of linearity had already been observed, in the operadic setting, see [11], Section 4).

The signatures we consider here are much more general than the signatures in [7], and cover the signatures appearing in [16, 12]. Note however that the latter works treat also non-wellfounded syntax, an aspect which we do not consider at all.

In our next work, we will propose a treatment of equational semantics for the present syntaxes. This approach should also be accommodated to deal with typed languages as done for elementary signatures in [17, 18, 2], or to model operational semantics as done for elementary signatures in [1].

3 The big category of modules

Modules over monads and the associated notion of linear natural transformation intend to capture the notion of "algebraic structure which is well-behaved with respect to substitution". An introduction on this subject can be found in our papers [13, 14]. Let us recall here the very basic idea.

Let R be a monad over a base category C. A module over R with range in a category D is a functor $M\colon \mathsf{C} \to \mathsf{D}$ endowed with an action of R, i.e., a natural "substitution" transformation $\rho\colon M \cdot R \longrightarrow M$ compatible with the substitution of R in the obvious sense. Given two modules M, N over the same monad and with the same range, a linear natural transformation $\phi\colon M \to N$ is a natural transformation of functors which is compatible with the actions in the obvious sense. This gives a category $\mathrm{Mod}^{\mathsf{D}}(R)$ of modules with fixed base R and range D.

It is useful for the present paper to consider a larger category which collects modules over different monads. For the following definition, we fix a range category D.

Definition 3.1 (The big module category)**.** We define the big module category $\mathrm{BMod}^{\mathsf{D}}_{\mathsf{C}}$ as follows:

- its objects are pairs (R, M) of a monad R on C and an R-module M with range in D.
- a morphism from (R, M) to (S, N) is a pair (f, m) where $f\colon R \longrightarrow S$ is a morphism of monads, and $m\colon M \longrightarrow f^*N$ is a morphism of R-modules (here f^*N is the functor N equipped with the obvious structure of R-module).

4 The category of arities

In this section, we give our new notion of arity. The destiny of an arity is to have representations in monads. A representation of an arity a in a monad R should be a morphism between two modules

[1] http://web.math.unifi.it/users/maggesi/strengthened/.

$\mathrm{dom}(a,R)$ and $\mathrm{codom}(a,R)$. For instance, in the case of the arity a of a binary operation, we have $\mathrm{dom}(a,R) := R^2$ and $\mathrm{codom}(a,R) := R$. Hence an arity should consist of two halves, each of which assigns to each monad R a module over R in a functorial way. However, in all our natural examples, we have $\mathrm{codom}(a,R) = R$ as above. Although this will no longer be the case in the typed case (which we do not consider here), we choose to restrict our attention to arities of this kind, where $\mathrm{codom}(a,R)$ is R.

From now on we will consider only monads over the category Set and modules with range Set. For technical reasons, see Section 7, we restrict our attention to the category of ω-cocontinuous endofunctors that we will denote $\mathrm{End}^\omega(\mathsf{Set})$. Analogously we will write Mon^ω (resp. BMod^ω) for the full subcategory of monads (resp. of modules over these monads) which are ω-cocontinuous.

We recall that finite limits commute with filtered colimits in Set. It follows that $\mathrm{End}^\omega(\mathsf{Set})$ has finite limits and arbitrary (small) colimits. This is the key ingredient in the proofs of ω-cocontinuity for most of our functors.

Definition 4.1 (Arities)**.** An *arity* is a right-inverse functor to the forgetful functor from the category BMod^ω to the category Mon^ω.

Now we give our basic examples of arities:

- Every monad R is itself a R-module. The assignment $R \mapsto R$ gives an arity which we denote by Θ.
- The assignment $R \mapsto *_R$, where $*_R$ denotes the final module over R is an arity which we denote by $*$.
- Given two arities a and b, the assignment $R \mapsto a(R) \times b(R)$ is an arity which we denote by $a \times b$. In particular $\Theta^2 = \Theta \times \Theta$ is the arity of any (first-order) binary operation and, in general Θ^n is the arity of n-ary operations.
- Given an endofunctor F of Set, we consider the *derived* functor given by $F' \colon X \mapsto F(X + *)$. It can be checked how when F is a module so is F'. Given an arity a, the assignment $R \mapsto a(R)'$ is an arity which we denote a' and is called *derivative* of a.
- Derivation can be iterated. We denote by $a^{(n)}$ the n-th derivative of a. Hence, in particular, we have $a^{(0)} = a$, $a^{(1)} = a'$, $a^{(2)} = a''$.
- For each sequence of non-negative integers $s = (s_1, \dots, s_n)$, the assignment $R \mapsto R^{(s_1)} \times \cdots \times R^{(s_n)}$ is an arity which we denote by $\Theta^{(s)}$. Arities of the form $\Theta^{(s)}$ are said *algebraic*. These algebraic arities are those which appear in [7].
- Given two arities a, b their composition $a \cdot b := R \mapsto a(R) \cdot b(R)$ is an arity.

Definition 4.2. A morphism among two arities $a_1, a_2 \colon \mathrm{Mon}^\omega \longrightarrow \mathrm{BMod}^\omega$ is a natural transformation $m \colon a_1 \longrightarrow a_2$ which, post-composed with the projection $\mathrm{BMod}^\omega \longrightarrow \mathrm{Mon}^\omega$, becomes the identity. We easily check that arities form a subcategory Ar of the category of functors from Mon^ω to BMod^ω.

Now we give two examples of morphisms of arities:

- The natural transformation $\mu \colon \Theta \cdot \Theta \longrightarrow \Theta$ induced by the structural composition of monads is a morphism of arities.
- The two natural transformations $\Theta \cdot \eta$ and $\eta \cdot \Theta$ from Θ to $\Theta \cdot \Theta$ are morphisms of arities.

Theorem 4.3. *The category of arities has finite limits and arbitrary (small) colimits.*

5 Categories of representations

Definition 5.1 (Signatures)**.** We define a signature $\Sigma = (O, \alpha)$ to be a family of arities $\alpha \colon O \longrightarrow \mathrm{Ar}$. A signature is said to be algebraic if it consists of algebraic arities.

Definition 5.2 (Representation of an arity, of a signature)**.** Given an ω-cocontinuous monad R over Set, we define a representation of the arity a in R to be a module morphism from $a(R)$ to R; a representation of a signature Σ in R consists of a representation in R for each arity in Σ.

Example 5.3. The usual $\mathsf{app}\colon \Lambda^2 \longrightarrow \Lambda$ is a representation of the arity Θ^2 into the monad Λ of λ-calculus 8.

Definition 5.4. Given a signature $\Sigma = (O, \alpha)$, we build the category Mon^Σ of representations of Σ as follows. Its objects are ω-cocontinuous monads equipped with a representation of Σ. A morphism m from (M, r) to (N, s) is a morphism of monads from M to N compatible with the representations in the sense that, for each o in O, the following diagram of M-modules commutes:

$$\begin{array}{ccc} \alpha_o(M) & \xrightarrow{r_o} & M \\ {\scriptstyle a_o(m)}\downarrow & & \downarrow{\scriptstyle m} \\ m^*(\alpha_o(N)) & \xrightarrow[m^* s_o]{} & m^*N \end{array}$$

where the horizontal arrows come from the representations and the left vertical arrow comes from the functoriality of arities and $m\colon M \longrightarrow m^*N$ is the morphism of monad seen as morphism of M-modules.

These morphisms, together with the obvious composition, turn Mon^Σ into a category which comes equipped with a forgetful functor to the category of monads.

We are primarily interested in the existence of an initial object in this category Mon^Σ.

Definition 5.5. A signature Σ is said representable if the category Mon^Σ has an initial object, which we denote $\hat{\Sigma}$.

Theorem 5.6. *Algebraic signatures are representable.*

For more details we refer to our paper [13] (Theorems 1 and 2). We give below a more general result (Theorem 7.8).

6 Modularity and the big category of representations

It has been stressed in [12] that the standard approach (via algebras) to higher-order syntax lacks modularity. In the present section we show in which sense our approach via modules enjoys modularity. The key for this modularity is what we call the big category of representations.

Suppose that we have a signature $\Sigma = (O, a)$ and two subsignatures Σ_1 and Σ_2 covering Σ in the obvious sense, and let Σ_0 be the intersection of Σ_1 and Σ_2. Suppose that these four signatures are representable (for instance because Σ is algebraic or strengthened in the sense defined below). Modularity would mean that the corresponding diagram of monads

$$\begin{array}{ccc} \hat{\Sigma}_0 & \longrightarrow & \hat{\Sigma}_1 \\ \downarrow & & \downarrow \\ \hat{\Sigma}_2 & \longrightarrow & \hat{\Sigma} \end{array}$$

is a pushout. The observation of [12] is that this diagram of raw monads is, in general, not a pushout. Since we do not want to change the monads, in order to claim for modularity, we will have to consider

a category of enhanced monads. Here by enriched monad, we mean a monad equipped with some additional structure, namely a representation of some signature.

Our solution to this problem goes through the following "big" category of representations, which we denote by RMon, where R may stand for representation or for rich:

- An object of RMon is a triple (R,Σ,r) where R is a monad, Σ a signature, and r is a representation of Σ in R.
- A morphism in RMon from $(R_1,(O_1,a_1),r_1)$ to $(R_2,(O_2,a_2),r_2)$ consists of an injective map $i := O_1 \longrightarrow O_2$ compatible with a_1 and a_2 and a morphism m from (R_1,r_1) to $(R_2,i^*(r_2))$, where $i^*(r_2)$ should be understood as the restriction of the representation r_2 to the subsignature (O_1,a_1) where we pose $i^*(r_2)(o) := r_2(i(o))$.
- It is easily checked that the obvious composition turns RMon into a category.

Now for each signature Σ, we have an obvious functor from Mon^Σ to RMon, through which we may see $\hat{\Sigma}$ as an object in RMon. Furthermore, an injection $i\colon \Sigma_1 \longrightarrow \Sigma_2$ obviously yields a morphism $i_* := \hat{\Sigma}_1 \longrightarrow \hat{\Sigma}_2$ in RMon. Hence our 'pushout' square of signatures as described above yields a square in RMon. The proof of the following statement is straightforward.

Modularity holds in RMon, in the sense that given a 'pushout' square of representable signatures as described above, the associated square in RMon is a pushout again.

As usual, we will denote by RMon^ω the full subcategory of RMon constituted by ω-cocontinuous functors. It is easy to check that the previous statement is equally valid in RMon^ω. Indeed, recall that, by our definition, the initial representation of representable signatures lies in RMon^ω.

7 Strengthening signatures

Guided by the ideas of Matthes and Uustalu [16] we introduce in our framework the notion of *strengthened arity*. For a category C, let us denote by $\mathrm{End}^\omega_*(\mathsf{C})$ the category of ω-cocontinuous *pointed endofunctors*, i.e., the category of pairs (F,η) of an ω-cocontinuous endofunctor F of C and a natural transformation $\eta\colon I \longrightarrow F$ from the identity endofunctor to F. A morphism of pointed endofunctors $f\colon (F_1,\eta_1) \longrightarrow (F_2,\eta_2)$ is a natural transformation $f\colon F_1 \longrightarrow F_2$ satisfying $f\circ\eta_1 = \eta_2$.

Definition 7.1. A *strengthened arity* is a pair (H,θ) where H is an ω-cocontinuous endofunctor of $\mathrm{End}^\omega(\mathsf{Set})$ (i.e., $H \in \mathrm{End}^\omega(\mathrm{End}^\omega(\mathsf{Set}))$) and θ is a natural transformation $\theta\colon H(-)\cdot\sim \longrightarrow H(-\cdot\sim)$ (where $H(-)\cdot\sim$ and $H(-\cdot\sim)$ have to be understood as functors from $\mathrm{End}^\omega(\mathsf{Set})\times\mathrm{End}^\omega_*(\mathsf{Set})$ to $\mathrm{End}^\omega(\mathsf{Set})$) satisfying $\theta_{X,(I,1_I)} = 1_{HX}$ and such that the following diagram is commutative

$$\begin{array}{ccc} H(X)\cdot Z_1\cdot Z_2 & \xrightarrow{\theta_{X,(Z_1\cdot Z_2,e_1\cdot e_2)}} & H(X\cdot Z_1\cdot Z_2) \\ & {\scriptstyle\theta_{X,(Z_1,e_1)}Z_2}\searrow \quad \nearrow{\scriptstyle\theta_{X\cdot Z_1,(Z_2,e_2)}} & \\ & H(X\cdot Z_1)\cdot Z_2 & \end{array} \tag{1}$$

for every endofunctor X and pointed endofunctors (Z_1,e_1), (Z_2,e_2). We refer to θ as the *strength* on H.

Our first task is to make clear that our wording is consistent in the sense that a strengthened arity H somehow yields a genuine arity $\tilde{H}$. For this task, for each monad R we pose $\tilde{H}(R) := H(R)$ and we exhibit on it a structure of R-module. We do even slightly more by upgrading H into a *module transformer* in the following sense:

Definition 7.2. A module transformer is an endofunctor of the big module category BMod^ω which commutes with the structural forgetful functor $\mathrm{BMod}^\omega \longrightarrow \mathrm{Mon}^\omega$.

Let (H,θ) be a strengthened arity. For every ω-cocontinuous monad R and ω-cocontinuous R-module M, we define the natural transformation $\rho^{H(M)}\colon H(M)\cdot R \longrightarrow H(M)$ as the composition $H(\rho^M)\cdot\theta_{M,R}$. Then $(H(M),\rho^{H(M)})$ is an R-module, and this construction upgrades H into a module transformer denoted by $\hat{H}$.

We call the restricion $\tilde{H}$ of the module transformer $\hat{H}$ to the category of monads the arity associated to the strengthened arity H.

Our next task is to upgrade our favorite examples of arities into strengthened arities:

- The arity Θ comes from the strengthened arity (H,θ) where H and θ are the relevant identities.
- The arity $*$ comes from the strengthened arity (H,θ) where H is the final endofunctor and θ is the relevant identity. This is the final strengthened arity.
- The arity $\Theta\cdot\Theta$ comes from the strengthened arity (H,θ) where $H := X \mapsto X\cdot X$ and $\theta_{X,Y} : X\cdot X\cdot Y \longrightarrow X\cdot Y\cdot X\cdot Y := X\cdot\eta^Y\cdot X\cdot Y$; here we have written η^Y for the morphism from the identity functor to Y (remember that Y is pointed).
- If an arity comes from a strengthened arity, so does its derivative (see Proposition 7.4).

Then we show how our basic constructions in the category of arities carries over the category of strengthened arities. First we describe this category. Its objects are strengthened arities and we take for morphisms from (H_1,θ_1) to (H_2,θ_2) those natural transformations $m\colon H_1 \longrightarrow H_2$ which are compatible with θ_1 and θ_2, that is, the diagram

$$\begin{array}{ccc} H_1(X)\cdot Z & \xrightarrow{\theta_1} & H_1(X\cdot Z) \\ {\scriptstyle m_X Z}\downarrow & & \downarrow{\scriptstyle m_{X\cdot Z}} \\ H_2(X)\cdot Z & \xrightarrow[\theta_2]{} & H_2(X\cdot Z) \end{array}$$

is commutative for every endofunctor X and every pointed endofunctor Z.

Theorem 7.3. *The category of strengthened arities has finite limits and arbitrary colimits.*

Next, we take care of the derivation. We denote by D the endofunctor of Set given by $A \mapsto A+*$. For any other pointed endofunctor X over Set we have a natural transformation $w^X\colon D\cdot X \longrightarrow X\cdot D$ given by

$$w^X_A\colon X(A)+* \longrightarrow X(A+*) \qquad w^X_A := X(i_A)+\eta_{A+*}\cdot\underline{*}$$

where $i_A\colon A \longrightarrow A+*$ and $\underline{*}\colon * \longrightarrow A+*$ are the inclusion maps.

Proposition 7.4. *If (H,θ) is a strengthened arity, then the pair (H',θ'), where $H' := X \mapsto H(X)'$ and $\theta'_{X,Z} := \theta_{X,Z}D\cdot H(X)w^Z$, is a strengthened arity. We call it the* derivative *of (H,θ).*

Now we point out the possibility of composing strengthened arities.

Definition 7.5. If $H := (H,\rho)$ and $K := (K,\sigma)$ are two strengthened arities, their composition $H\cdot K$ is the pair $(H\cdot K,\theta)$ where θ is defined by $\theta_{X,(Z,e)} := H(\sigma_{X,(Z,e)})\cdot\rho_{K(X),(Z,e)}$.

Proposition 7.6. *This composition turns strengthened arities into a strict monoidal category.*

Next, we turn to the main interest of strengthened arities (or signatures) which is that the fixed point we are interested in inherits a structure of monad.

Lemma 7.7. *Let (H,θ) be a strengthened arity. Then the fixed point T of the functor $F := I+H$ is ω-cocontinuous and comes equipped with a structure of $\tilde{H}$-representation which is the initial object in the category of the $\tilde{H}$-representations.*

We say that a signature is strengthened if it is a family of strengthened arities. The previous lemma leads immediately to the following result.

Theorem 7.8. *Strengthened signatures are representable.*

8 Examples of strengthened syntax

Lambda-calculus modulo α-equivalence One paradigmatic example of syntax with binding is the λ-calculus. We denote by $\Lambda(X)$ the set of lambda-terms up to α-equivalence with free variables 'indexed' by the set X. It is well-known [6, 3, 13] that Λ has a natural structure of cocontinuous monad where the monad composition is given by variable substitution.

It can be easily verified that application $\mathsf{app}\colon \Lambda^2 \longrightarrow \Lambda$ and abstraction $\mathsf{abs}\colon \Lambda' \longrightarrow \Lambda$ are Λ-linear natural transformations, that is, Λ is a monad endowed with a representation ρ of the signature $\Sigma = \{\mathsf{app}\colon \Theta^2, \mathsf{abs}\colon \Theta'\}$. The monad Λ is initial in the category Mon^Σ of ω-cocontinuous monads equipped with a representation of the signature Σ.

This is an example of algebraic signature and thus already treated by other previous works [13, 14, 7]. Here we simply remark that our new theory covers such a classical case.

Explicit composition operator We now consider our first example of non-algebraic signature. On any monad R, we have the composition operator (also called join operator) $\mu^R\colon R\cdot R \longrightarrow R$ which has arity $\Theta\cdot\Theta$. We will refer to the μ^R operator as the *implicit* composition operator. An interesting problem is to see if this kind of operators admits a corresponding *explicit* version, i.e., if they can be implemented as a syntactic construction. As we have seen before $\Theta\cdot\Theta$ is a strengthened arity hence we can build syntaxes with explicit composition operator of kind

$$\mathsf{join}\colon \Theta\cdot\Theta \longrightarrow \Theta.$$

Of course, this is only a *syntactic* composition operator, in the sense that it does not enjoy several desirable conversion rules like associativity, two-side identity and the obvious compatibility rules with the other syntactic constructions present in the signature. In our next work we will show how to construct such kind of *semantic* composition operator.

Let us mention that given a monad R, the unit $\eta_R\colon I \longrightarrow R$ is not an R-linear morphism (in fact, I is not even an R-module in general). For this reason we cannot treat examples of syntax with explicit unit.

Syntax and semantics with explicit substitution On any monad R, we have a series of substitution operators $\sigma_n\colon R^{(n)}\cdot R^n \longrightarrow R$ which simultaneously replace n formal arguments in a term with n given terms. As observed by Ghani and Uustalu [12], these substitution morphisms satisfy a series of compatibility relations which mean that they come from a single morphism $\mathsf{subst}\colon C \longrightarrow \Theta$ where C is identified as the coend

$$C = \int^{A:\mathsf{Fin}} \Theta^{(A)} \times \Theta^A.$$

Here Fin stands for a skeleton of the category of finite sets, Θ^A denotes the cartesian power and $\Theta^{(A)}$ is defined by $\Theta^{(A)}(R, X) := R(X + A)$. Since coends are special colimits, and strengthened arities admit colimits, we just have to check that the bifunctorial arity $(A, B) \mapsto \Theta^{(A)} \times \Theta^B$ factors through the category of strengthened arities. As far as objects are concerned, this follows from our results in Section 7. The verification of the compatibility of the corresponding "renaming" and "projection" morphisms with the strengthened structures is straightforward.

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