

Modeling Terms by Graphs with Structure Constraints (Two Illustrations)

Clemens Grabmayer

Gran Sasso Science Institute

Viale F. Crispi 7, 67100 L'Aquila AQ, Italy

clemens.grabmayer@gssi.it

In the talk at the workshop my aim was to demonstrate the usefulness of graph techniques for tackling problems that have been studied predominantly as problems on the term level: increasing sharing in functional programs, and addressing questions about Milner's process semantics for regular expressions. For both situations an approach that is based on modeling terms by graphs with structure constraints has turned out to be fruitful. In this extended abstract I describe the underlying problems, give references, provide examples, indicate the chosen approaches, and compare the initial situations as well as the results that have been obtained, and some results that are being developed at present.

1 Introduction

For my talk at the workshop I prepared two examples from my past and current work that highlight the usefulness and the potential of graph techniques for problems that have been approached predominantly as questions about terms: increasing sharing in functional programs, and tackling problems about Milner's process semantics for regular expressions. The unifying element of these two illustrations consists in modeling terms by term graphs or transition graphs with structure constraints (higher-order features or labelings with added conditions), and in being able to go back and forth between terms and graphs.

The first illustration, which I only touched on in my talk, concerns the definition, and the efficient implementation of maximal sharing for the higher-order terms in the λ -calculus with letrec. For solving this problem, Jan Rochel and I developed a representation pipeline from terms via higher-order term graphs and first-order term graphs to deterministic finite-state automata.

The setting for the second illustration, on which I focused in my presentation, is Milner's process semantics of regular expressions, which yields nondeterministic finite-state automata (NFAs) whose equality is studied under bisimilarity. In my current work with Wan Fokkink, I use labelings of process graphs that witness direct expressibility by a regular expression via a condition on the graph topology.

My motivation for explaining these two cases together developed as follows. While working on problems concerning the process semantics of regular expressions I have repeatedly benefited from the previous work on modeling cyclic λ -terms by structure-constrained term graphs. It turned out that many concepts and methods that Jan Rochel and I had developed could be adapted in order to define structure-constrained process graphs that directly represent regular expressions under the process semantics. It seemed worthwhile to compare the settings and the results so that the flow of ideas from one setting to the other, and probably back, might become clearer. Perhaps this can be of help in similar situations.

In this extended abstract I explain the setting and the background of the underlying problems, provide references, give examples, and informally describe the chosen approaches: in Section 2, for the implementation of maximal sharing of functional programs, and in Section 3, for the problems concerning the process semantics of regular expressions. In order to highlight differences, and to identify similarities

that enabled a transferal of ideas between the two illustrations, I compare them in Section 4 with respect to the initial situation, the desired concepts, and the defined structure-constrained graphs.

2 Maximal sharing of functional programs

The first example concerns the definition, and the efficient implementation of maximal sharing for functional programs, and more specifically, for the higher-order terms in the λ -calculus with letrec [13].

Graph representations of terms in the λ -calculus with letrec are crucial for the implementation of functional programming languages, in particular for facilitating the efficient execution of compiled programs in sharing-graph form via graph reduction. However, these graph representations were never conceived as term graph representations that keep their intended meaning under bisimilarity. In fact they do not behave well under bisimilarity with respect to the unfolding semantics of terms in the λ -calculus with letrec. In order to study the compactification of functional programs (in their usual language), Jan Rochel and I therefore looked for term graph representations that support compactification under bisimilarity while preserving the intended meaning, and being easy to compute and to translate back into terms. Our focus on these desiderata (see also Figure 9 later) led us to structure-constrained term graph representations, for which we investigated a number of different options [12]. We eventually defined classes of ' λ -higher-order-term-graphs' and of ' λ -term-graphs' that are closed under functional bisimilarity and have natural correspondences with the terms in the λ -calculus with letrec (see again in Figure 9).

On this basis Jan Rochel and I developed a 'representation pipeline' from higher-order terms to deterministic finite-state automata (DFAs): (1) Terms in the λ -calculus with letrec can be represented by appropriately defined higher-order term graphs, which are first-order term graphs together with higher-order features such as a scope function, or an abstraction prefix function, that are defined on the set of vertices (see [12]); (2) higher-order term graphs are encoded as first-order term graphs (see also [12]), and (3) first-order term graphs are represented as DFAs (see [13]). In this way unfolding equivalence on terms is represented by bisimulation equivalence on term graphs (higher-order and first-order), and ultimately, by language equivalence of DFAs. In [13] we also define a readback operation from DFAs that arise by the representation pipeline back to terms in the λ -calculus with letrec. This operation makes it possible to go back and forth between terms and representing DFAs: it has the property that the representation via (1), (2), and (3) is the inverse of the readback operation.

Figure 1 and Figure 2 provide an example for the translation of a term in the λ -calculus with letrec into higher-order and first-order graph representations, and eventually to a finite-state automaton. Figure 1 covers the part from the syntax tree to λ -higher-order-term-graphs, and Figure 2 the remaining part via a λ -term-graph and an 'incomplete λ -DFA' to a ' λ -DFA'.

In Figure 1 we start from the syntax tree of the term, model the recursive definition by a recursive backlink, replace variable names by nameless dummies that have binding backlinks to the corresponding abstraction vertices, and draw scopes. In this way we obtain first-order term graphs with scope sets that satisfy the conditions for scope sets in the concept of 'higher-order term graph' by Blom [6]. We call the specific version obtained here a λ -higher-order term graph *with scope sets*. In doing so we distinguish it from a λ -higher-order term graph *with an abstraction prefix function*, where scopes of abstraction vertices are recorded per vertex v via the stack of those abstraction vertices in whose scope v resides. See both versions of λ -higher-order term graph for the example here at the bottom of Figure 1.

In Figure 2 we start from the λ -higher-order-term-graph obtained in Figure 1, and crucially encode all scope information (recorded by the scope set, or by the abstraction prefix function) by introducing a scope vertex for the single edge in this example that crosses the boundary of a scope. We call the resulting

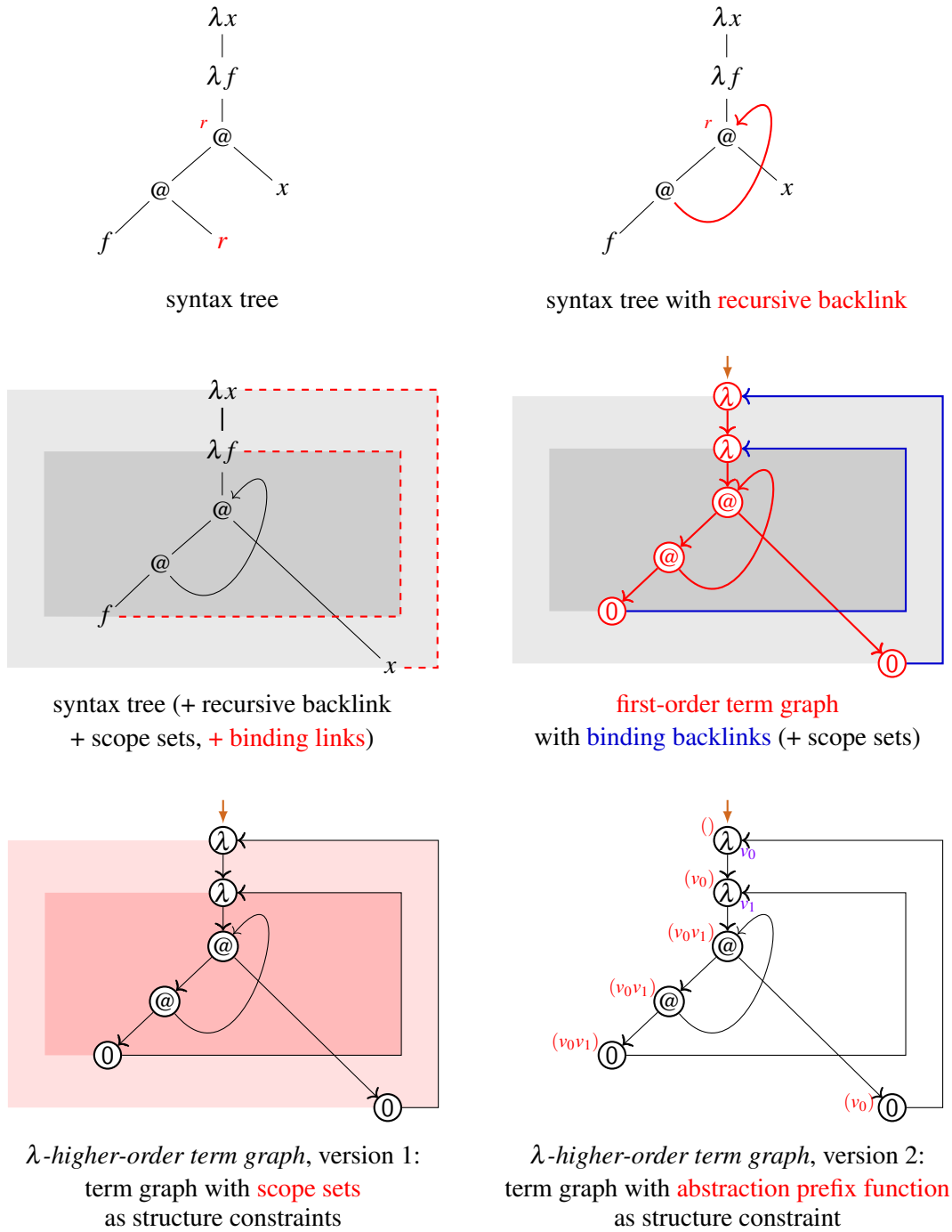


Figure 1: Stepwise translation of the term $\lambda x. \lambda f. \text{let } r = f r x \text{ in } r$ in the λ -calculus with letrec via the construction of its syntax tree, and its modification into a first-order term graph with scope sets to obtain a λ -higher-order term graph in one of two versions: a higher-order term graph with scope sets for abstraction nodes, and with an abstraction-prefix function on the set of vertices.

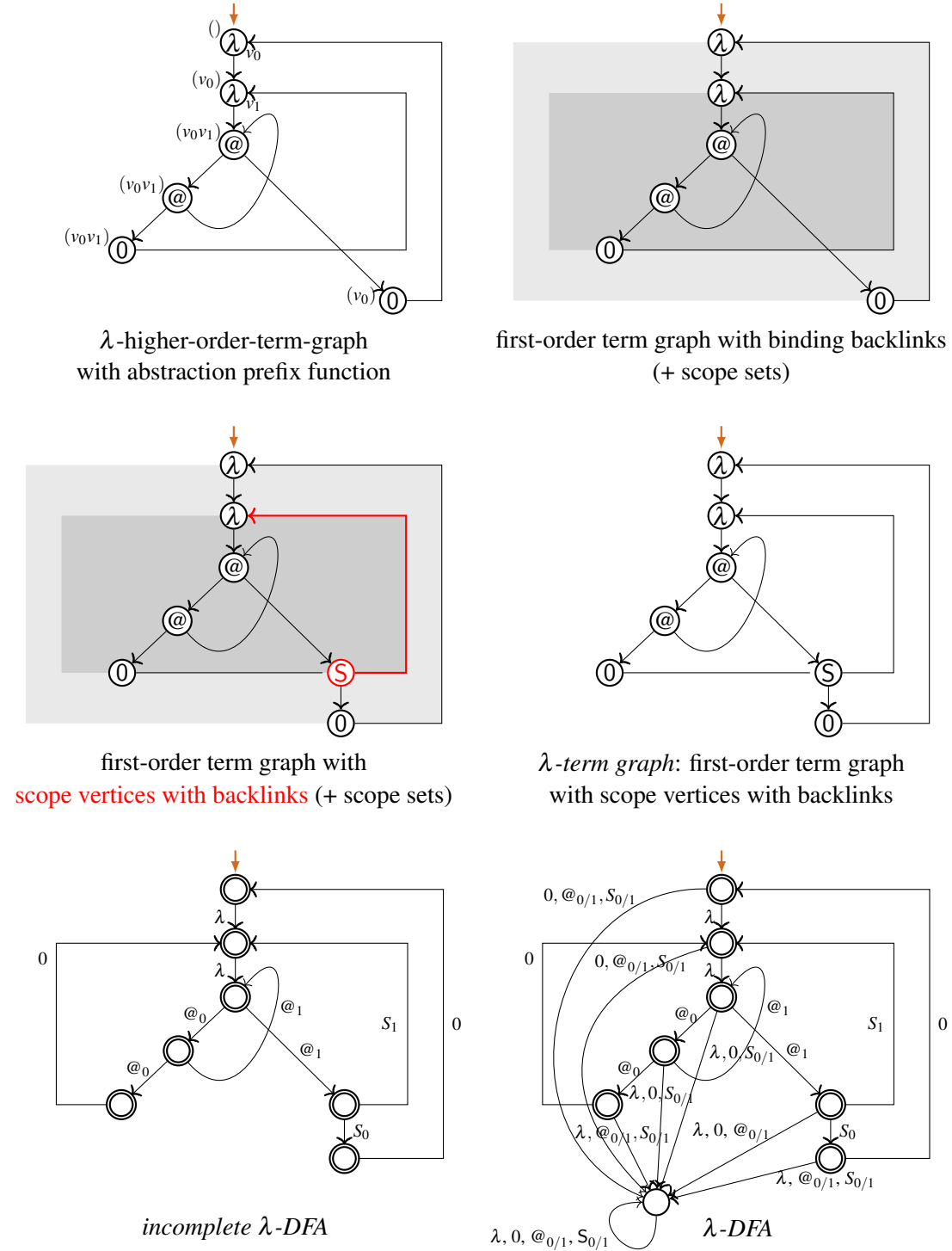


Figure 2: Stepwise translation of the term $\lambda x.\lambda f.\text{let } r = frx \text{ in } r$ in the λ -calculus with letrec from the λ -higher-order term graph obtained in Fig. 1 via a λ -term graph (a first-order term graph in which ends of scopes are encoded by scope vertices) and via an incomplete λ -DFA into a λ -DFA. In the last step a non-accepting state is added to the incomplete λ -DFA to which all missing transitions are directed.

first-order term graph a λ -term-graph. By using an intuitive correspondence of term graphs with DFAs, we translate this first-order term graph further to obtain an incomplete λ -DFA and eventually a λ -DFA, both of which represent the term $\lambda x. \lambda f. \text{let } r = f r x \text{ in } r$ from which we started.

Via the correspondence statements on which the representation pipeline is based, unfolding equivalence of terms in the λ -calculus with letrec can be computed in pseudo-quadratic time $O(n^2 \cdot \alpha(n))$ where α is the inverse Ackermann function (see [13]). Again via the correspondences described above, via DFA-minimization, and via the readback a maximally shared form of higher-order terms can be computed in $O(n^2 \cdot \log n)$ time (again see [13]).

In order to demonstrate the maximal-sharing method as a manageable optimizing transformation for the compilation of functional programs, we developed the software tool [17] that is available on Haskell's Hackage platform. Following the definition of maximally shared representations via the representation pipeline in [13] (see also Rochel's thesis [18] for more context), this tool transforms a given functional program in the λ -calculus with letrec (the basis of the Core Language of the Glasgow Haskell Compiler) into a λ -term-graph, and then into a λ -DFA. It prints intermediate representations textually, and displays the obtained incomplete λ -DFA graphically. The λ -DFA is then minimized, and a maximally shared representation of the original program is computed by the readback operation as the result.

Together with Vincent van Oostrom, I have set out to generalize this technique of representing higher-order terms as term graphs with added features that are needed for modeling scopes of binding constructs. But rather than capturing the constraints on the term graph structure by 'ad hoc' features, we now used 'nesting' as the single added structuring concept. In [14] we defined, and investigated the behavioral semantics of 'nested term graphs' that arise as follows: by nesting first-order term graphs into the vertices of, initially, a first-order term graph, and then of nested term graphs that have already been formed.

3 Process semantics of regular expressions

The second illustration concerns the process semantics of regular expressions. Milner developed a complete axiomatization of bisimulation equivalence for finite process graphs represented in μ -term notation [16] (1984). On this basis he turned to descriptions of finite process graphs by regular expressions with a unary star operation.¹ Also in [16] he defined a semantics $\llbracket \cdot \rrbracket_P$ for regular expressions as finite-state processes: 0 is interpreted as the deadlock process, 1 as the immediately terminating process, letters as actions that lead to termination, and the symbols '+', '.', and $(\cdot)^*$ as operators that enable choice between processes, sequential composition of processes, and iteration of a process, respectively. See Figure 4 for two examples of process interpretations of regular expressions via $\llbracket \cdot \rrbracket_P$. Formally, Milner's definition of $\llbracket \cdot \rrbracket_P$ yields finite process graphs by an inductive definition on the structure of regular expressions.

A close variant $\llbracket \cdot \rrbracket'_P$ of Milner's process semantics $\llbracket \cdot \rrbracket_P$ has later been defined via a transition system specification (TSS): the TSS \mathcal{T} in Figure 3 explains the operational behavior of a regular expression (the option to do a labeled step, or to terminate) inductively for each of the constants and letters, and for each of the operators. This TSS is an adaptation for regular expressions with a unary star operation of a TSS that was formulated for regular expressions with a binary star operation by Bergstra, Bethke, and Ponse [5] (1994). By means of the TSS \mathcal{T} the set $\text{RegExp}(A)$ of regular expressions over a given set A of action labels is endowed with the structure of a labeled transition system (LTS) $\mathcal{L}(\mathcal{T})$: there is an a -transition from e_1 to e_2 in $\mathcal{L}(\mathcal{T})$ if and only if $e_1 \xrightarrow{a} e_2$ is provable in \mathcal{T} . Then the variant process interpretation $\llbracket e \rrbracket'_P$ of a regular expression e is defined within this encompassing LTS $\mathcal{L}(\mathcal{T})$ on

¹While regular expressions with a binary star operation were introduced by Kleene in [15] (1951), regular expressions with a unary star operation seem to have been first formulated by Copi, Elgot, and Wright [8] (1958).

$$\begin{array}{c}
\frac{}{1 \downarrow} \quad \frac{e_i \downarrow}{(e_1 + e_2) \downarrow} \quad \frac{e_1 \downarrow \quad e_2 \downarrow}{(e_1 \cdot e_2) \downarrow} \quad \frac{}{(e^*) \downarrow} \\
\frac{a \xrightarrow{a} 1}{e_1 + e_2 \xrightarrow{a} e'_i} \quad \frac{e_i \xrightarrow{a} e'_i}{e_1 \cdot e_2 \xrightarrow{a} e'_1 \cdot e_2} \quad \frac{e_1 \downarrow \quad e_2 \xrightarrow{a} e'_2}{e_1 \cdot e_2 \xrightarrow{a} e'_2} \quad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*}
\end{array}$$

Figure 3: Transition system specification \mathcal{T} of computations enabled by regular expressions.

$RegExp(A)$ as the LTS, or process graph, that consists of the part of $\mathcal{L}(\mathcal{T})$ that is reachable from e . This process graph $\llbracket e \rrbracket'_P$ can be shown to be finite for every regular expression e . It is closely related, and in fact always bisimilar to the interpretation $\llbracket e \rrbracket_P$ of e according to Milner's process semantics $\llbracket \cdot \rrbracket_P$.

Every labeled transition system with a finite set of vertices can be construed as a non-deterministic finite-state automaton (NFA). Therefore the process semantics $\llbracket \cdot \rrbracket_P$ for regular expressions can be viewed as a translation into NFAs whose equality is studied with respect to bisimilarity, rather than with respect to language equivalence. Indeed, Antimirov [2] (1996) arrived at the same automaton-translation for regular expressions, but without process theory and bisimulation equivalence in mind. He pursued the goal of obtaining for a given regular expression e , in a natural way, an NFA that accepts the language $L(e)$ denoted by e , and that is smaller than NFAs accepting $L(e)$ that are obtained by classical algorithms for the translation of regular expressions into NFAs. For this purpose he introduced, for regular expressions $e \in RegExp(A)$ the set of 'partial derivatives' $\partial_a(e)$ of e with respect to letters $a \in A$, and a termination predicate $tm(e)$. More precisely, he gave definitions by induction on the structure of regular expressions for the functions:

$$\begin{array}{ll}
\partial_a(\cdot) : A \times RegExp(A) \longrightarrow \mathcal{P}(RegExp(A)) & tm : RegExp(A) \longrightarrow \{0, 1\} \subseteq \mathbb{N} \\
\langle a, e \rangle \longmapsto \partial_a(e), & e \longmapsto tm(e).
\end{array}$$

in such a way that the following correspondences hold with respect to the transition system \mathcal{T} :

$$\partial_a(e) = \{e' \in RegExp(A) \mid \vdash_{\mathcal{T}} e \xrightarrow{a} e'\}, \quad tm(e) = \begin{cases} 1 & \text{if } \vdash_{\mathcal{T}} e \downarrow, \\ 0 & \text{otherwise.} \end{cases}$$

In this way the NFA that is obtained by repeated applications of Antimirov's partial derivatives to a regular expression e coincides with the NFA that corresponds to the LTS $\llbracket e \rrbracket'_P$ as obtained by the TSS \mathcal{T} . That NFA is in turn bisimilar (as a consequence of bisimilarity of the LTSs involved as mentioned above) to the NFA that corresponds to the interpretation $\llbracket e \rrbracket_P$ of e in Milner's process semantics.

Unlike for the standard language semantics $\llbracket \cdot \rrbracket_L$, not every NFA can be expressed by a regular expression under the process interpretation $\llbracket \cdot \rrbracket_P$. That is, not every NFA is bisimilar to the process translation NFA of some regular expression. This is witnessed by the two examples in Figure 5, both of which were suggested already by Milner. He showed, in [16], that the three-vertex example without termination in Figure 5 is not $\llbracket \cdot \rrbracket_P$ -expressible. That the second example with two termination-permitting vertices in Figure 5 is not $\llbracket \cdot \rrbracket_P$ -expressible was proved by Bosscher [7].

Still in [16], Milner adapted the complete axiomatization by Salomaa [19] for language equivalence of regular expressions. He started from a version of Salomaa's system in which all product expressions in the axioms and rules are commuted, see Figure 6. The rule Fix is subject to the 'non-algebraic' side-condition that the regular expression e does not have the 'empty word property', that is, the language

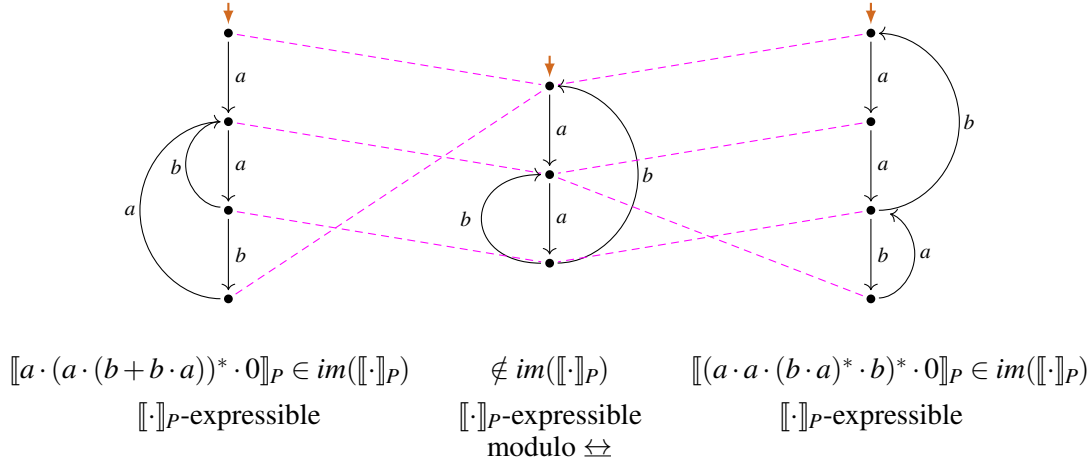


Figure 4: Process graphs that are expressible by regular expressions via the process semantics $\llbracket \cdot \rrbracket_P$, and expressible modulo bisimilarity \Leftrightarrow . The graph on the left is the process semantic of $a \cdot (a \cdot (b + b \cdot a))^* \cdot 0$, the one on the right of $(a \cdot a \cdot (b \cdot a)^* \cdot b)^* \cdot 0$. These graphs are bisimilar, as shown here via bisimulations with their bisimulation collapse, a process graph that is not the process semantic of a regular expression.



Figure 5: Process graphs that are neither $\llbracket \cdot \rrbracket_P$ -expressible nor $\llbracket \cdot \rrbracket_P$ -expressible modulo bisimilarity \Leftrightarrow . In the process graph on the left, both vertices permit immediate termination (indicated by the outer circles).

(B_1)	$e + (f + g) = (e + f) + g$	(B_7)	$e \cdot 0^* = e$
(B_2)	$(e \cdot f) \cdot g = e \cdot (f \cdot g)$	(B_8)	$e \cdot 0 = 0$
(B_3)	$e + f = f + e$	(B_9)	$e + 0 = e$
(B_4)	$(e + f) \cdot g = e \cdot g + f \cdot g$	(B_{10})	$e^* = 0^* + e \cdot e^*$
(B_5)	$e \cdot (f + g) = e \cdot f + e \cdot g$	(B_{11})	$e^* = (0^* + e)^*$
(B_6)	$e + e = e$	(A_8)	$0 \cdot e = 0$
(Refl)	$e = e$		

$\frac{e = f}{f = e} \text{Symm}$	$\frac{e = f \quad f = g}{e = g} \text{Trans}$	$\frac{e = f}{C[e] = C[f]} \text{Cxt}$	$\frac{e = f \cdot e + g}{e = f^* \cdot g} \text{Fix (if } f \text{ does not have e.w.p.)}$
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Figure 6: Complete axiomatization of equality of regular expressions under the language semantics $\llbracket \cdot \rrbracket_L$. The system is due to Aanderaa, and corresponds to Salomaa's system by commuting product expressions. Axiom (A_8) from Salomaa's system is derivable, and not part of Aanderaa's system. Axioms that are not sound under the process semantics $\llbracket \cdot \rrbracket_P$ are colored in red. Milner's axiomatization $\text{BPA}_{0,1}^*$ of bisimilarity of regular expressions under the process semantics $\llbracket \cdot \rrbracket_P$ arises by dropping the unsound axioms (in red).

interpretation $\llbracket e \rrbracket_L$ of e does not contain the empty word. This system is close to the complete axiomatization for language equivalence that was presented by Aanderaa [1] independently from Salomaa's work (Aanderaa's system was probably not directly known to Milner). Milner dropped the two rules from the system that are unsound under the process semantics (left-distributivity B_5 , and the axiom B_8), but additionally took up the axiom (A_8) from Salomaa's original system, which describes a correct interaction property of 0 as deadlock with process concatenation. The resulting system is sound for the process semantics $\llbracket \cdot \rrbracket_P$. It has later been called $\text{BPA}_{0,1}^*$ as an adaptation of Basic Process Algebra BPA to regular expressions as terms that describe process behavior with respect to $\llbracket \cdot \rrbracket_P$.

Milner noticed that completeness for $\text{BPA}_{0,1}^*$ cannot be settled directly by Salomaa's arguments. This is due to the incompleteness modulo bisimilarity \Leftrightarrow of the image of the process semantics $\llbracket \cdot \rrbracket_P$. That namely implies that not every finite regular system of equations is solvable by a regular expression (for example, specifications that correspond to the process graphs in Figure 5 are not solvable). However, being able to solve arbitrary finite regular systems of equations by regular expressions is a crucial lemma in Salomaa's and Aanderaa's completeness proofs. Recognizing this difficulty, Milner formulated the question as to whether $\text{BPA}_{0,1}^*$ is indeed a complete axiomatization for bisimilarity of interpretations of regular expressions in the process semantics $\llbracket \cdot \rrbracket_P$. In addition, he also formulated the problem of characterizing those process graphs that are bisimilar to process interpretations of regular expressions, and a star-height problem for regular expressions over a single-letter alphabet.

The known approaches to these questions by Milner fall, broadly speaking, into two groups that are distinguished by how they model processes that are represented by regular expressions: either by working with process terms whose operational semantics is governed by structural operational semantics (SOS) rules, such as TSSs, or by reasoning about regular recursive process specifications of a certain structure. Taking a new approach, I have set out to use structure-constrained process graphs, see below.

Building on work from the process term tradition, Fokkink (1996-97) showed that the restriction of Milner's system to exit-less iteration, which he called 'perpetual-loop' and 'terminal cycle', is complete for the general case with 'empty' 1-steps [10], and for the easier case without [11]. To achieve this result he completely overturned Salomaa's and Aanderaa's proof technique of extension of terms (obtaining a common extension for semantically equal terms) into its contrary, a strategy of term minimization.

Also working with term calculi for process terms, Corradini, De Nicola, and Labella [9] define a subclass of regular expressions, those without occurrences of 0 that satisfy the 'hereditary non-empty word property (hnwep)', and give a '(purely) equational' axiomatization for $\llbracket \cdot \rrbracket_P$ on regular expressions with these restrictions. Indeed their result shows that Milner's axiomatization without the axioms involving 0 is complete for regular expressions from that class. This is because for regular expressions with hnwep the non-equational side-condition on the fixed-point rule Fix is irrelevant, and therefore can be dropped, which turns the axiomatization into a purely equational one.

Regular expressions that may contain 0, but satisfy the property hnwep of Corradini, De Nicola, and Labella can be characterized as follows: for no iteration subexpression f^* of e does $\llbracket f \rrbracket_P$ proceed to a process p such that: p has the option to immediately terminate, and p has the option to do a proper step, and terminate later. Motivated by this, I call these expressions '1-return-less(-under-*)'. They turned out to be relevant in my current work on structure-constrained process graphs, see below.

Using recursive specifications to formalize processes that are induced by regular expressions, Baeten and Corradini (2005) introduced 'well-behaved specifications' [3]. These systems of equations are arranged according to trees with back-bindings ('palm trees') with a 'loop-exit' structure requirement. This concept enabled Baeten, Corradini, and myself to show that expressibility modulo bisimilarity of a finite process graph by a regular expression is decidable [4], although via a super-exponential procedure.

My current approach to the axiomatization problem (in work with Wan Fokkink) takes the conscious

step to reasoning about process graphs for which the palm-tree form is relaxed significantly as constraint. A crucial step is the formulation of a concept of transition graph labeling that is inspired by Milner’s notion of ‘loop’. Transitions (action-labeled edges) are decorated by additional marker labels that witness that the syntax tree of a regular expression can be inscribed on to a (typically cyclic) process graph. In this way a labeling witnesses that the process graph can be expressed *directly* by a regular expression. This opens the way to develop bisimilarity-preserving transformations of directly expressible process graphs, in order to constructively connect any two given directly expressible process graphs that are bisimilar.

Figure 10 in Section 4 gathers the initial motivation for defining structure-constrained process graphs, and puts the desiderata here in the context of the properties of Milner’s process semantics $\llbracket \cdot \rrbracket_P$. It also gives a preliminary overview on results that are being developed at the moment.

By modifying a concept introduced by Milner in [16], we call a process graph a ‘loop’ if all paths from the start vertex return to it, and termination is only permitted at the start vertex. A ‘loop subgraph’ in a process graph G is a loop that is generated from a vertex v of G by a set T of ‘loop-entry transitions’ from v as follows: the subgraph of G that consists of all vertices and edges that are reachable on paths departing from v via an edge in T until v is reached again. Furthermore we call ‘loop elimination’ a procedure that, starting from a given process graph repeatedly identifies a loop subgraph, drops its loop-entry transitions, and performs garbage collection (removing vertices and edges that have become unreachable from the start vertex). We say that a process graph G satisfies the *loop existence and elimination condition (LEE)* if by loop elimination from G a process graph without an infinite behavior (that is, without an infinite trace) can be reached.

Figure 7 in its upper row shows two loop elimination steps that are performed starting from the process graph in the middle of Figure 4. These steps lead to a process graph without any transitions, and hence without an infinite trace. Thus they witness that the original process graph has the property LEE. By contrast, none of the two process graphs in Figure 5 contains a loop subgraph: the two-vertex graph does not because the termination condition of a loop would be violated; and the three-vertex graph does not because no transition from a vertex v generates a subchart in which all infinite paths return to v . Hence these process graphs, which are not $\llbracket \cdot \rrbracket_P$ -expressible modulo \leftrightarrow , do not satisfy the property LEE.

In its lower row, Figure 7 records a procedure of reassembly of the process graph in the upper left corner from the results that have been obtained during loop elimination. Thereby an approximation of the original process graph is assembled that is structured by 1-transitions. We call it a *structured LEE-witness*. Figure 8 indicates that a LEE-witness is obtained from the structured version by overlaying the separately recorded loop subgraphs on to the original process graph, and by labeling the identified loop-entry transitions according to the order in which they have been removed during loop elimination.

A LEE-witness records the loop elimination procedure in a process graph by marking transitions that have been recognized as loop-entry transitions with a label that indicates its number (or nesting depth) in the procedure. It is subject to conditions that follow from this intuition, and the requirement that loop elimination leads to a process graph without an infinite trace. Thus a LEE-witness is a labeling of a process graph that is subject to appropriate conditions that witnesses that the graph satisfies LEE. In this way we obtain a class of structure-constrained process graphs that consists of all graphs that have a LEE-witness, and hence satisfy LEE. The arising class properly extends the class of process graphs that are the process semantics of some regular expression: the process graph in the middle of Figure 4 has a LEE-witness, and satisfies LEE (see Figure 7 and Figure 8), but it is not $\llbracket \cdot \rrbracket_P$ -expressible.

The concept of LEE-witness is an important technical tool for investigating transformations between process graphs that satisfy the graph-topological property LEE, and for extracting regular expressions from such process graphs. It facilitates a number of results such as the following: (1) LEE is preserved under functional bisimilarity \Rightarrow for process graphs without empty steps. The proof of this statement relies

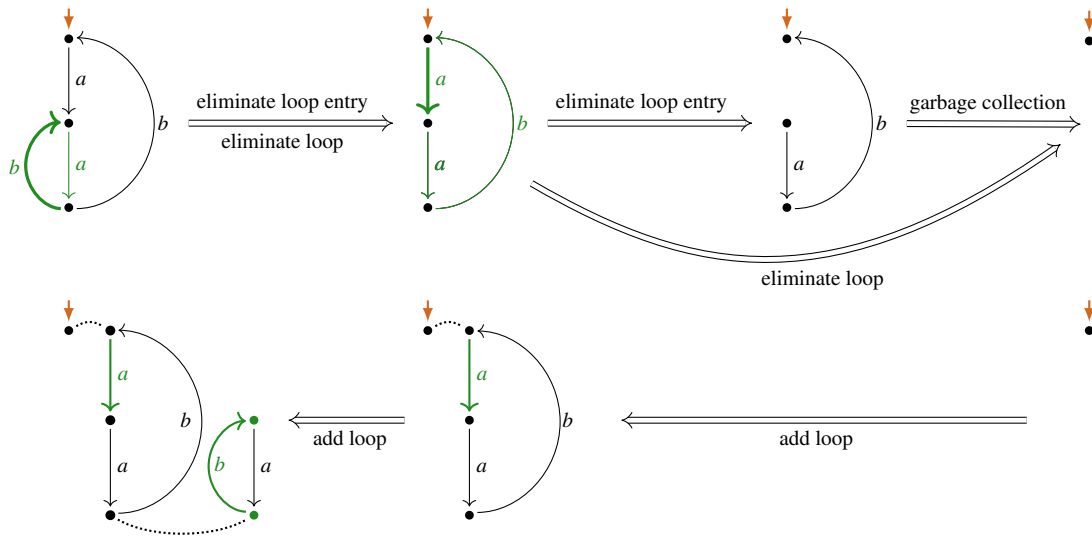


Figure 7: Loop elimination for the left process graph in the upper row by repeatedly identifying a loop-entry transition, then removing it, and performing garbage collection. Since a process graph without infinite behavior is reached, the original process graph has the property LEE. In the second row a structured version of the original graph is reassembled in converse direction by using the eliminated loops.

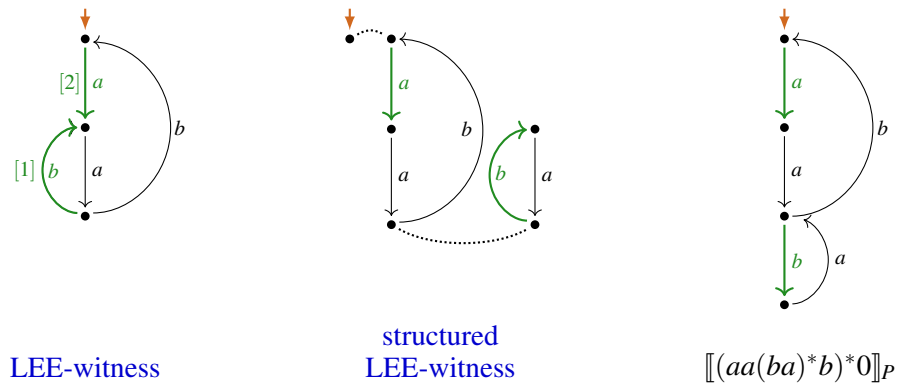


Figure 8: A LEE-witness for the original process graph in Fig. 7 is obtained by overlaying the loops from the structured version that has been obtained by loop-addition synthesis in Fig. 7, and by number-labels that record the order of loop removal. The structured form of the LEE-witness indicates a correspondence with the process semantics of one of the regular expressions considered in Fig. 4.

on the fact that LEE-witnesses can be transferred along functional bisimulations. (2) From every process graph G without 1-transitions that satisfies LEE a 1-return-less regular expression e can be extracted for which $\llbracket e \rrbracket_P \simeq G$ holds, that is, such that e expresses G under $\llbracket \cdot \rrbracket_P$ modulo bisimilarity. This statement can be proved by using the number labels of the loop-entry transitions in a LEE-witness to define a bottom-up extraction procedure of a regular expression.

These statements lead to a new partial answer to Milner's question about how $\llbracket \cdot \rrbracket_P$ -expressibility of finite process graphs can be characterized: A finite process graph G is $\llbracket \cdot \rrbracket_P$ -expressible by a 1-return-less regular expression if and only if the bisimulation collapse of G satisfies the property LEE.

4 Comparison desiderata and results

Apart from demonstrating the usefulness of working with structure-constrained graphs, another motivating aim for my talk was to obtain a clearer view of the similarity and the difference of the two situations. In particular I wanted to understand why I was able to benefit from a flow of ideas from the first to the second illustration. As a first step towards a better understanding I assembled, for each of the two settings, a list of the motivations and desiderata for graph representations arising from the initial problems, and of the results that have been obtained, or that are being developed. These overviews are gathered in Figure 9 and in Figure 10.

The initial situations are markedly different: a graph semantics that is studied under bisimilarity is

λ -calculus with letrec with respect to the *unfolding semantics*

Known: graph representations of terms in the λ -calculus with letrec are used in compilers of functional languages. However:

- these graph representations were not intended for use under transformations that involve bisimilarity \simeq , and do not behave well under such transformations.

Aim: a term graph semantics that:

- ▷ has a natural correspondence with terms in λ -calculus with letrec,
- ▷ supports compactification under bisimilarity \simeq ,
- ▷ permits efficient operations to translate between terms to graphs.

Defined: *Structure-constrained term graphs* as a semantics for terms in the λ -calculus with letrec:

- ▶ the class \mathcal{H} of higher-order λ -term graphs, with interpretation function $\llbracket \cdot \rrbracket_{\mathcal{H}}$,
- ▶ the class \mathcal{F} of first-order λ -term graphs, with interpretation function $\llbracket \cdot \rrbracket_{\mathcal{F}}$.

They have the following properties:

- (i) λ -term graphs are first-order term graph encodings of λ -higher-order term graphs,
- (ii)** \mathcal{H} and \mathcal{F} are closed under functional bisimilarity \Rightarrow (and hence under collapse),
- (iii)** there is a back-/forth correspondence with terms in the λ -calculus with letrec such that:
 - there are efficient translation and readback operations (computable in $O(n^2 \log n)$ and $O(n \log n)$ time),
 - the translation is the inverse of the readback.

Figure 9: Motivation for developing structure-constrained term graph representations for the first illustration, the λ -calculus with letrec; and an overview of the obtained concepts and results. The key results (ii) and (iii) are highlighted as they correspond to analogous results for the second illustration, see Fig. 10.

Regular expressions with respect to the process semantics

Given: Milner’s process graph semantics $\llbracket \cdot \rrbracket_P$ was designed for study under bisimilarity \Leftrightarrow .

However, the semantics $\llbracket \cdot \rrbracket_P$ has some peculiar properties:

- the image of $\llbracket \cdot \rrbracket_P$ is not closed under functional bisimilarity \Rightarrow
- the image of $\llbracket \cdot \rrbracket_P$ is incomplete modulo bisimilarity \Leftrightarrow

Aim: in order to tackle completeness of Milner’s axiomatization, and the recognizability of $\llbracket \cdot \rrbracket_P$ -expressibility modulo \Leftrightarrow , it is desirable to:

- ▷ reason with (‘sufficiently many’) graphs that are $\llbracket \cdot \rrbracket_P$ -expressible modulo \Leftrightarrow ;
- ▷ understand incompleteness modulo \Leftrightarrow by a structural graph property.

Defined / under construction / current aim: *Structure-constrained process graphs*, in particular:

- ▶ the class of finite process graphs with the property LEE which consists of all those process graphs that have a (layered) LEE-witness labeling.

It has the following properties:

- (i) it extends the image of the process semantics $\llbracket \cdot \rrbracket_P$;
- (ii)** it is closed under functional bisimilarity \Rightarrow (and hence under bisimulation collapse) in the special case of the absence of 1-transitions (empty-step transitions);
- (iii)** it permits efficient back and forth translations to and from 1-return-less expressions;
- (iv) it characterizes $\llbracket \cdot \rrbracket_P$ -expressibility modulo \Leftrightarrow by a 1-return-less regular expression of a graph’s collapse: a finite process graph G is $\llbracket \cdot \rrbracket_P$ -expressible modulo \Leftrightarrow by a 1-return-less regular expression if and only if the bisimulation collapse of G satisfies LEE.

Figure 10: Motivation for developing structure-constrained process graphs for the second illustration, the process semantics for regular expressions; and an overview of the results that we are currently working out. The key results (ii) and (iii) are emphasized with their labels in boldface in order to highlight their correspondence with the analogous results (ii) and (iii) for the first illustration in Fig. 9.

provided by Milner’s process semantics of regular expressions, whereas graph representations for cyclic λ -terms that are used in compilers do not behave well under bisimilarity. For representing cyclic λ -terms an appropriate class of term graph representations needed to be defined, for example one based on Blom’s higher-order term graphs [6]. Yet also the incompleteness under functional bisimilarity of the image of the process semantics stimulated extending this class of graphs to one with more satisfying properties.

The joining element of the results obtained in the two settings consists in the definition of classes of structure-constrained graphs that, on the one hand, are closed under functional bisimilarity (and hence are closed under the operation of taking the bisimulation collapse), and that, on the other hand, enable a natural, and efficiently computable correspondence with the class of terms that is relevant for the setting. This observation is highlighted in Figure 9 and Figure 10 by the items with boldface numbers: (ii) for closedness under functional bisimilarity \Rightarrow , and (iii) for the natural correspondence with terms.

In conclusion I want to repeat a request that I have put to the participants of the workshop: I am interested in, and would like to hear about, other situations and settings in which structure-constrained graph representations might be useful, or have already been developed and used successfully.

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References

- [1] Stål Aanderaa (1965): *On the Algebra of Regular Expressions*. Technical Report, Applied Mathematics, Harvard University.
- [2] Valentin Antimirov (1996): *Partial Derivatives of Regular Expressions and Finite Automaton Constructions*. *Theoretical Computer Science* 155(2), pp. 291–319, doi:10.1016/0304-3975(95)00182-4.
- [3] J.C.M. Baeten & F. Corradini (2005): *Regular Expressions in Process Algebra*. In: *Proceedings of LICS 2005*, IEEE Computer Society 2005, pp. 12–19, doi:10.1109/LICS.2005.43.
- [4] J.C.M. Baeten, F. Corradini & C.A. Grabmayer (2007): *A Characterization of Regular Expressions Under Bisimulation*. *Journal of the ACM* 54(2), doi:10.1145/1219092.1219094.
- [5] J.A. Bergstra, I. Bethke & A. Ponse (1994): *Process algebra with iteration and nesting*. *The Computer Journal* 37(4), p. 243258, doi:10.1093/comjnl/37.4.243.
- [6] Stefan Blom (2001): *Term Graph Rewriting, Syntax and Semantics*. Ph.D. thesis, Vrije Universiteit Amsterdam.
- [7] D. J. B. Bosscher (1997): *Grammars Modulo Bisimulation*. Ph.D. thesis, Universiteit van Amsterdam.
- [8] Irving M. Copi, Calvin C. Elgot & Jesse B. Wright (1958): *Realization of Events by Logical Nets*. *Journal of the Association For Computing Machinery* 5(2), doi:10.1007/978-1-4613-8177-8_1. Also in Stephen L. Bloom (editor): ‘Calvin C. Elgot: Selected Papers’, p. 1–16, Springer, 1982.
- [9] Flavio Corradini, Rocco De Nicola & Anna Labella (2002): *An Equational Axiomatization of Bisimulation over Regular Expressions*. *Journal of Logic and Computation* 12(2), pp. 301–320, doi:10.1093/logcom/12.2.301.
- [10] W.J. Fokkink (1996): *An Axiomatization for the Terminal Cycle*. Technical Report 167, Utrecht University, Logic Group Preprint Series. Available at <http://bit.ly/2uJVEDF> (on semanticscholar.com).
- [11] W.J. Fokkink (1997): *Axiomatizations for the perpetual loop in process algebra*. In P. Degano, R. Gorrieri & A. Marchetti-Spaccamela, editors: *Proceedings of the 24th Colloquium on Automata, Languages and Programming - ICALP’97, Bologna, LNCS 1256*, Springer, pp. 571–581, doi:10.1007/3-540-63165-8_212.
- [12] Clemens Grabmayer & Jan Rochel (2013): *Term Graph Representations for Cyclic Lambda Terms*. In: *Proceedings of TERMGRAPH 2013, EPTCS 110*, pp. 56–73, doi:10.4204/EPTCS.110. ArXiv:1302.6338v1.
- [13] Clemens Grabmayer & Jan Rochel (2014): *Maximal Sharing in the Lambda Calculus with Letrec*. In: *Proceedings of the 19th ACM SIGPLAN International Conference on Functional Programming, ICFP ’14*, ACM, New York, NY, USA, pp. 67–80, doi:10.1145/2628136.2628148.
- [14] Clemens Grabmayer & Vincent van Oostrom (2015): *Nested Term Graphs*. In Aart Middeldorp & Femke van Raamsdonk, editors: *Post-Proceedings of TERMGRAPH 2014, Vienna, Austria, July 13, 2014, EPTCS 183*, Open Publishing Association, pp. 48–65, doi:10.4204/EPTCS.183.4. ArXived at:1405.6380v2.
- [15] S.C. Kleene (1951): *Representation of Events in Nerve Nets and Finite Automata*. Technical Report, U.S. Air Force Project RAND Research Memorandum RM-704, doi:<http://dx.doi.org/10.1515/9781400882618-002>.
- [16] Robin Milner (1984): *A Complete Inference System for a Class of Regular Behaviours*. *Journal of Computer and System Sciences* 28(3), pp. 439 – 466, doi:10.1016/0022-0000(84)90023-0.
- [17] Jan Rochel & Clemens Grabmayer (2014): *Maximal Sharing in the Lambda Calculus with letrec*. Haskell Implementation of the method of [13], available at <http://hackage.haskell.org/package/maxsharing/>.
- [18] Jan Rochel (2016): *Unfolding Semantics of the Untyped λ -Calculus with letrec*. Ph.D. thesis, Utrecht University. Defended on June 20, 2016. Available at <http://rochel.info/thesis/thesis.pdf>.
- [19] Arto Salomaa (1966): *Two Complete Axiom Systems for the Algebra of Regular Events*. *Journal of the ACM* 13(1), pp. 158–169, doi:10.1145/321312.321326.