

# Reasoning about Emergence of Collective Memory

R. Ramanujam

Institute of Mathematical Sciences

Homi Bhabha National Institute

Chennai, India

jam@imsc.res.in

We offer a very simple model of how collective memory may form. Agents keep signalling within neighbourhoods, and depending on how many support each signal, some signals “win” in that neighbourhood. By agents interacting between different neighbourhoods, ‘influence’ spreads and sometimes, a collective signal emerges. We propose a logic in which we can reason about such emergence of memory and present preliminary technical results on the logic.

## 1 Introduction

*Strictly speaking, there is no such thing as collective memory – part of the same family of spurious notions as collective guilt. But there is collective instruction ... All memory is individual, unreproducible; it dies with each person. What is called collective memory is not a remembering but a stipulating: that this is important, and this is the story about how it happened, with the pictures that lock the story in our minds.*

Susan Sontag ([24])

Any discussion on individual values and social values visits the question, *How are we to act?* at some point. The question, of course, is, who is this **we** referred to here? Clearly this **we** is a social construction, one that depends on the very social norms and social values that we wish to reason about ([27]). Integral to such social construction of a collective is the memory ascribed to that collective. Group identity is constructed structurally by ascribing memory to the group, and in turn, such identity shapes its memory. Remembrance has a crucial impact on preferences and values, influences action.

It is here that Susan Sontag’s quote above assumes significance. Sontag calls collective memory a process of stipulation. Somehow the collective ascribes importance to an item of memory, authenticates it and symbolizes it; then on, the symbolism “locks” the memory item, in Sontag’s account.

Note that this is a significant departure from the structural conception of memory, that visualises memory as a notebook, and remembering as looking it up. Wittgenstein ([28]) strongly attacked such a conception of memory.

I saw this man years ago: now I have seen him again, I recognize him, I remember his name. And why does there have to be a cause of this remembering in my nervous system? Why must something or other, whatever it may be, be stored up there in any form? Why must a trace have been left behind? Why should there not be a psychological regularity to which no physiological regularity corresponds?

If this upsets our concept of causality then it is high time it was upset.

Scholars like Sutton ([25]) have discussed this issue at length. For Wittgenstein, social acts were important in shaping memory, and based on this, scholars like Rusu ([23]) even talk of *social time*, and modern theories of connectionism and distributed memory build on many such notions.

For us, these remarks are relevant from two viewpoints. The 1950's saw the development of automata theory as a study of *memory structures*, and in theory of computation, automata provide a model of memory that Wittgenstein might have approved of. In this view, memory is not a table to be looked up, but is constituted by states of being of the automaton. Observations cause changes in state, some states remember (some of the past) and some forget. Thus, remembering and forgetting are built into system structure. Such a view is important for seeing memory and reasoning as *interdependent* rather than as separate (as psychologists used to consider). Logicians are used to equating automata and logics, as in the case of monadic second order logics of order or in the case of Pressburger arithmetic. (Wittgenstein would have approved.)

The other viewpoint relates to *distributed memory*, where interacting agents rely on memory external to them. Computer science has evolved impressive models of highly flexible interaction and memory that has literally changed the everyday life of much of humanity in the last few decades. Today, memory storage on the "cloud" has become indispensable for many, and people voluntarily 'post' personal information to make it socially available in an attempt to write personal information into social memory.

In social theory, the notion of collective memory is influential. *Maurice Halbwachs* ([14]) talked of how an individual's understanding of the past is strongly linked to a group consciousness, which in turn is a form of *group memory* that lives beyond the memories of individuals that form the group.

For the logician, these notions pose an interesting challenge: what are the logical properties of collective remembering? What is the rationale followed by a group in ascribing / stipulating collective importance to events and their remembering? Why is a particular idealisation chosen? These are difficult questions to answer, but a more modest reformulation of such questions offers an approach to solutions. If the memory of an automaton is describable via logic, we can perhaps build a model of group and individual memory based on automata whose interactions lead to collective memory, which in turn influences behaviour of individual automata.

Why should one bother? In ([19]) Rohit Parikh speaks of *cultural structures* providing an infrastructure to social algorithms (much as data structures do for computational algorithms). Epistemic reasoning is an essential component of social algorithms, as persuasively argued by Parikh. We can then see collective memory as an essential gradient of its infrastructure creating the 'common ground' in which social objectives and communications are interpreted ([10]). Moreover, social algorithms such as elections need social memory if they are to achieve their democratic purpose.

Moreover, there has been extensive research in recent years on notions of collective agency ([26]), collective action ([22]), collective belief ([13]) and many more. However, the memory required for collective action, belief and agency is largely assumed rather than explicitly discussed. While social theorists extensively discuss the role of society's needs for remembrance (as a way of acknowledging the past and taking responsibility for it) the notion of memory as an infrastructural need for such functions is often glossed over. Moreover, the notions of memory that inform these discussions are embodied in text, icons and physical tokens, much like the notebook that Wittgenstein refers to (and objects to). Contemporary reality, with the extension of human memory using technology, suggests that more dynamic, behavioural models of memory, and the re-inforcing behaviour that leads to stipulation as referred to by Sontag, may be relevant.

What follows is a very simple, perhaps very simplistic, attempt at formalization of this notion, inspired by the study of *population protocols* in distributed computing ([4]) and large anonymous games [11]. We offer this formal model tentatively, as an initial step of a (hopefully) detailed research programme. As it stands, the model has no agency or epistemic attitudes or social aggregation. Instead it focusses only on how local attempts at signalling "importance" of an observation can spread and lead to some stable phenomenon that can be meaningfully construed as collective memory.

The crucial element here is that individuals perceive events differently, based on social and cultural background. The word *partition* would evoke the image of equivalence classes to a combinatorist in general, but very likely, that of a terrible tragedy first to an Indian combinatorist.

## 1.1 Related work

The literature on memory studies principally consists of two strands, one on individual memory, substantially incorporated into psychology, and the other on group memory, primarily on social representations of history ([7]). A major question of interest is whether such groups manifest *emergent*, robustly collective forms of memory. In general, while social context is seen to influence remembering, the act of remembering is held to be individual. However the literature on collective intentionality ([27]) considers collective memory as a form of collective attention to the past. The concepts underlying the model we discuss below are greatly inspired by the “*we-mode*”, discussed by the philosopher *Raimo Tuomela*, though we place an additional emphasis on patterns of social interaction (rather than the interactions themselves).

If we admit the existence of emergent collective memory, the major question is whether the processes of social collective memory resemble in any way the processes of individual memory or that of small groups (like families) studied extensively by psychologists. In an astonishing thesis from an interdisciplinary collaboration of neurobiology, medicine, cognitive science and anthropology, Anastasio et al ([2]) assert that these two processes are in fact the same. The model we present below, taking automata as memory representations, attempts to build a ‘social automaton’ (roughly speaking) in this spirit, showing a correspondence of processes.

The mechanism that we use in the model is closely related to that of *majority dynamics* ([15]) and *spread of influence* ([18]) studied in the analysis of social networks. There are also influential logical studies of diffusion in social networks ([5]). In other papers, we have studied similar phenomena in the context of *large games* ([20], [21]). While there is a correspondence at an intuitive level between memory systems and large games, a formal correspondence would lead to many potential applications.

The logic we propose here is a simple linear time temporal logic (with past) with variables and bounded quantification. There is extensive logical literature on the role of memory in multi-agent epistemic reasoning. For instance, the notion of bounded-recall plans has been studied in ([12], [8], [1]), to mention some recent works. However, this literature is principally on the interactions between individual memories; our point of departure is in studying emergent collective memory as an entity in itself. Moreover, the logic we discuss here is extremely poor in logical machinery, in comparison: there is no agency and no ascription of epistemic attitudes. Rather, we discuss only the temporal evolution of signalling interaction and the stability of signals. On the other hand, logical studies of social networks such as ([5]) and [17]) are closer in spirit to the automaton model we present, though the logics themselves are different.

## 2 A model

Let  $N$  denote a fixed finite set of agent names. Let  $\mathcal{C} \subseteq 2^N$  be a nonempty set of nonempty subsets of  $N$ , referred to as *neighbourhoods* over  $N$ . We assume  $|N| > 2$ .

For presenting the model we will make some simplifying assumptions. We fix a finite **signal alphabet**  $\Gamma$  common to all agents. Let  $\Gamma = \{\gamma_1, \dots, \gamma_m\}$ .

Let  $I \in \mathcal{C}$  and  $|I| = k$ . A **distribution** over  $I$  is an  $m$  tuple of integers  $\mathbf{y} = (y_1, \dots, y_m)$  such that

$y_j \geq 0$  and  $\sum_{j=1}^m y_j = k$ ,  $1 \leq j \leq m$ . That is, the  $j$ th component of  $\mathbf{y}$  gives the number of agents in the neighbourhood  $I$  who give signal  $\gamma_j$ . Let  $\mathbf{Y}[I]$  denote the set of all signal distributions of a neighbourhood  $I$  and let  $\mathbf{Y} = \bigcup_{I \in \mathcal{C}} \mathbf{Y}[I]$ .

The main idea is this. All agents initially receive an external input and assume some state. At each state, an agent produces a signal. Interactions occur in neighbourhoods nondeterministically, and an agent who is a member of many, could be interacting in different neighbourhoods (though every interaction at any instant is confined to one neighbourhood). Each interaction induces a state transition that is determined only by the distribution of signals: it does not depend on who is signalling what, but how many are producing each signal. Such interactions keep occurring repeatedly until a stable configuration is reached.

Below, for  $I \in \mathcal{C}$ , we use the notation  $\Gamma^I$  for a vector of signals, one signal for each of the agents in  $I$ . Note that every such vector induces a distribution over  $\Gamma$  in  $\mathbf{Y}[I]$ .

We will consider systems of agents below where the set of possible states  $Q$  is uniform for all agents. Thus a state transition in  $Q \times Q$  is also possible uniformly for all agents. By a triple  $(\gamma, q, q')$  we mean that an agent who is signalling  $\gamma$  changes state from  $q$  to  $q'$ .  $\sigma : \Gamma \rightarrow (Q \times Q)$  specifies such a signal based change of state. Let  $\Sigma$  denote the collection of such maps.

**Definition 1** A memory system over  $\mathcal{C}$  is a tuple  $M = (Q, \delta, \iota, \omega)$ , where

- $Q$  is a finite set of memory states,
- $\iota : N \rightarrow Q$  is the initial state,
- $\omega : Q \rightarrow \Gamma$  is the signalling function, and
- $\delta$  is a finite family of transition relations  $\delta_I \subseteq (\mathbf{Y}[I] \times \Sigma)$ , where  $I \in \mathcal{C}$ .

## 2.1 Dynamics

A configuration  $\chi$  is an element of  $Q^N$ . Let  $\omega(\chi)$  denote the vector in  $\Gamma^N$  induced by  $\omega$ . For  $I \in \mathcal{C}$  and let  $\mathbf{y}_I$  be a distribution of signals induced by the vector  $\omega(\chi)$  restricted to  $I$ .

**Definition 2** We say that an  $I$ -interaction is enabled at  $\chi$  if there is a transition  $(\mathbf{y}, \sigma)$  in  $\delta_I$  where  $\mathbf{y}$  is the distribution induced by  $\omega(\chi)$  and  $\sigma \in \Sigma$  is a signal-based change of state.

The effect of the transition is determined by  $\sigma$  and the new configuration  $\chi'$  is given by:

$$\chi'(j) = q', \text{ where } j \in I, \sigma(\omega(\chi(j))) = (\chi(j), q')$$

and  $\chi'(j) = \chi(j)$ , otherwise. Thus, we have a transition  $(\chi, I, \chi')$  on configurations labelled by neighbourhoods.

The dynamics of  $M$  is then given by a configuration graph  $G_M$  whose vertices are configurations and edges are labelled by neighbourhoods: an edge  $(\chi, I, \chi')$  is present if an  $I$ -interaction is enabled at  $\chi$  by a transition in  $\delta$  with resulting configuration  $\chi'$  as above. Note that  $\iota$  specifies an initial configuration  $\chi_0$ . A history  $\rho$  is any finite or infinite path in  $G_M$  starting from  $\chi_0$ . When  $\rho$  is finite,  $|\rho|$  denotes its length. Let  $\mathcal{H}_M$  denote the set of all maximal histories of  $M$ .

## 2.2 Collective memory in $M$

Consider a history  $\rho = \chi_0\chi_1\dots$  of system  $M$ . We say that signal  $\gamma$  is eventually stable for neighbourhood  $I$  in  $\rho$  if there exists  $k$  such that for all  $\ell \geq k$ , (when  $\rho$  is finite, for all  $\ell$  such that  $k \leq \ell \leq |\rho|$ ), and for all  $j \in I$ ,  $\omega(\chi_\ell(j)) = \gamma$ .

We say that  $\gamma$  is in collective memory in  $\rho$  if it is eventually stable for  $N$  in  $\rho$ : that is, no matter what interactions take place, all agents remain in states that emit signal  $\gamma$ .

When we have stable configurations, we see them as formation of collective memory. For this it is of course essential that a history allows for signalling to spread across neighbourhoods; if some neighbourhoods never interact, then signalling can remain confined within pockets. So we consider spanning histories where we impose the condition that every interaction that is infinitely often enabled (according to the transition rule) in an infinite history takes place infinitely often. This is a typical fairness condition used in the theory of computation, but weaker, or different, conditions may be sufficient for many systems. For instance, we might ask that the union of neighbourhoods in a history span all of  $N$ ; this merely says that all the agents have interacted at least once. Note that this depends on  $\delta$ : the distributions specified may already disable some agents or neighbourhoods from ever interacting.

**Definition 3** We say that the system  $M$  supports emergence of collective memory if, for every spanning history  $\rho$  of  $M$ , there exists a signal  $\gamma_\rho$  that is in collective memory in  $\rho$ .

## 2.3 An example

Consider a system with two signals  $\{g, b\}$ , standing for “good” and “bad”. There are only two states:  $G$  and  $B$ , signalling  $g$  and  $b$  respectively. Initially every agent perceives some global event as good or bad. Thus  $\iota$  is an arbitrary distribution of signals in the system.

The transition rule is simple: for any neighbourhood  $I$ , if more than half in  $I$  signal  $x$ , then all agents in  $I$  signal  $x$  in the new state. If they are exactly even, they continue evenly matched. Let  $I$  be a neighbourhood with  $k$  agents.  $\delta_I = \{(m, n), (g, q, G), (b, q, G) | m > n, m + n = k\} \cup \{(m, n), (g, q, B), (b, q, B) | m < n, m + n = k\} \cup \{(m, m), (s, q, q) | k = 2m, s \in \{g, b\}\}$ .

Now we can see that whether either of the signals becomes stable in a history depends on both the initial perception as well as which  $I$ -interactions are enabled. If odd-sized neighbourhoods can interact, a signal will begin to dominate. When the initial distribution is exactly even, and the interacting neighbourhoods are always split evenly, neither signal dominates the other.

On the other hand, suppose that a large fraction of the population receives the signal  $g$ , but  $\delta$  only enables neighbourhoods with the majority signalling  $b$  and a minority signalling  $g$  to interact. Then the signal  $b$  emerges as a stable signal, despite the initial distribution. This can be seen as the influence of social structures on collective memory.

Such behavioural analysis is common in the study of *runaway phenomena* ([6]) and so-called *informational cascades* ([9]).

## 2.4 Some subclasses

Consider a memory system in which all interactions are constrained to be pairwise. In such a system, the distribution profiles are entirely irrelevant, since given a pair of agents, there are four possible pairs of signals, determined by their states, and we only need to specify the resulting pair of states. Thus  $\delta \subseteq Q^4$ . Such systems have been studied as **population protocols** ([4]), for which a number of technical results are known.

We also have other interesting subclasses of systems: for instance, those where the transition relation is presented as  $\delta_k$  where  $1 < k \leq N$ . That is, only the size of a neighbourhood determines whether it can interact and not the identities of agents in it. (Note that even this restriction does not rule out the predatory dominance of a signal, as illustrated in the example above.)

Of great interest to social systems is where  $\mathcal{C}$  represents a hierarchy: we have an ordering  $<$  on  $N$  and impose the condition that all  $I \in \mathcal{C}$  are downward-closed with respect to the ordering relation.

However, note that these subclasses only constrain interaction structure and not the memory updates. This may be consonant with the discussion on collective memory in social theory, whereby social structures (and belief systems) constrain opportunity and influence, but what persists in social memory may well be impervious to social structures.

## 2.5 Computational power

Note that when  $|\Gamma| = d$ , every distribution over  $\Gamma$  is a  $d$ -dimensional vector in  $\mathbf{N}^d$ . The initial state  $\iota$  specifies such a distribution. Now consider a spanning history that is stably signalling  $\gamma$ ; we can consider this the output of the memory system for that history. Viewed thus, every system that supports emergence of collective memory can be said to *compute* a function from  $\mathbf{Y} \rightarrow \Gamma$ . Alternatively, we can consider such a function to be a predicate over  $\mathbf{N}^d \times \{1, \dots, d\}$ . Thus we can speak of arithmetical predicates computable by memory systems.

An important theorem in the study of population protocols guides us to the study of what memory systems can compute. In the study of population protocols, systems come with an *output function*, mapping to the two element output alphabet  $\{0, 1\}$  (without loss of generality). In this case we can consider the population protocol to be computing a predicate over  $\mathbf{N}^d$ .

Recall that a *semi-linear set* is a subset of  $\mathbf{N}^d$  that is a finite union of *linear* sets of the form  $\{\mathbf{b} + k_1 \mathbf{a}_1 + \dots + k_m \mathbf{a}_m \mid k_1, \dots, k_m \in \mathbf{N}\}$ , where  $\mathbf{b} \in \mathbf{N}^d$  and  $\mathbf{a}_1 \dots \mathbf{a}_m$  are  $d$ -dimensional basis vectors.

**Theorem 1 [3]:** *A predicate is computable by a population protocol iff it is semi-linear.*

An alternative characterization of these predicates is that they can be expressed in first-order Presburger arithmetic, which is first order arithmetic on the natural numbers with addition but not multiplication.

**Theorem 2** *Given a memory system  $M$ , checking whether  $M$  supports emergence of collective memory is decidable. Moreover the class of predicates computable by memory systems is exactly that of population protocols.*

There are two parts to the proof. We construct a Parikh automaton ([16]) that represents the configuration space of the memory system and reduce the check for stability of signals in the system to the nonemptiness problem for the associated Parikh automaton. This gives us the required decision procedure.

In the process we show that every predicate computed by a memory system is semi-linear. By the earlier theorem, such a predicate can be computed by a population protocol. Conversely, since population protocols are a subclass of memory systems, the predicates computed by the former are computable by memory systems as well. (There are some details related to the output function, and the restriction to spanning histories, which complicate the construction a little.)

To get an intuitive idea of the construction, we define Parikh automaton below, which in turn needs the definition of Presburger arithmetic.

Firstly, let  $\Delta = \{a_1, \dots, a_m\}$  be any finite alphabet, and  $w \in \Delta^*$ . The Parikh image of  $w$  counts the number of occurrences of each letter of the alphabet in  $w$ . Formally, we have the map  $\pi : \Delta^* \rightarrow \mathbf{N}^m$  given by:  $\pi(a_i) = e_i$  and  $\pi(uv) = \pi(u) + \pi(v)$ , where  $e_i$  is the unit vector of length  $m$  where the  $i^{\text{th}}$  coordinate is 1.

Clearly the map  $\pi$  can be lifted to alphabets of the form  $\Delta \times D$  where  $D \subseteq \mathbf{N}^d$ :  $\pi(a_i, \hat{k}) = \hat{k}$  and  $\pi(uv) = \pi(u) + \pi(v)$ . We can also consider the alphabetic projection into  $D^*$ :  $\lambda(a_i, \hat{k}) = a_i$  and  $\lambda(uv) = \lambda(u)\lambda(v)$ .

Presburger arithmetic is first order logic with the only atomic formulas of the form  $t \text{ rel } t'$  where  $t$  and  $t'$  are terms, and  $\text{rel} \in \{>, <, \geq, \leq\}$ . Terms are built from two constants 0 and 1, and variables, using addition and  $n \cdot t$  where  $n \in \mathbf{N}$ . Formulas are interpreted over the structure  $\mathcal{N} = (\mathbf{N}, +, \cdot, 0, 1)$ . When  $\phi(\hat{x})$  is a formula with free variables  $\hat{x}$  the notion  $\mathcal{N}, \hat{k} \models \phi(\hat{x})$  is defined in the standard fashion.

Given  $D \subseteq \mathbf{N}^d$  and  $L \subseteq (\Delta \times D)^*$ , and a formula  $\phi$  of Presburger arithmetic, we define  $L \upharpoonright_{\phi} = \{\lambda(w) \mid \mathcal{N}, \pi(w) \models \phi(\hat{x})\}$ .

**Definition 4** A **Parikh automaton** of dimension  $d > 0$  is a pair  $(A, \phi)$  where  $\phi(x_1, \dots, x_d)$  is a formula of Presburger arithmetic over  $d$  variables, and  $A$  is a finite word automaton with the finite alphabet  $\Delta \times D$  where  $D \subseteq \mathbf{N}^d$ . We say that  $(A, \phi)$  recognizes  $L(A, \phi) = L(A) \upharpoonright_{\phi}$ , where  $L(A)$  is the language recognized by the automaton  $A$ .

For the construction we need, there are some points to note. Configurations of memory systems carry states, from which we compute signal distributions which cause state changes. For the Parikh automaton, transitions are labelled by distributions. More importantly we need to carry the neighbourhood based signalling in the transitions of the Parikh automaton. While these are matters of detail, the harder part of the proof is the definition of the formula of Presburger arithmetic, for which we closely follow the proof method for population protocols ([3]).

### 3 A logic for the rationale

We now turn our attention to our principal logical interest, namely, the rationale by which agents decide what signals are chosen. This is surely complex, depending on the systems being modelled. Here we propose a minimal logic, in which agents evaluate signals based on their own evaluation of the current state and their evaluation of signals from the neighbourhood, depending on the signal distributions.

Let  $V$  be a countable set of variables. Let the terms of the logic be defined as

$$\tau ::= i \mid x, \quad i \in N, \quad x \in V$$

That is, a term is either an agent name or a variable (which takes agents as its values). Let  $\mathcal{P}$  denote a countable set of atomic propositional symbols.

The formulas of the logic are built using the following syntax:

$$\begin{aligned} \Phi ::= & \tau_1 = \tau_2 \mid \tau \in I \mid p @ \tau, p \in \mathcal{P} \mid \gamma @ \tau, \gamma \in \Gamma \mid \neg \phi \mid \\ & \phi_1 \vee \phi_2 \mid \ominus \phi \mid \bigcirc \phi \mid \diamond \phi \mid \heartsuit \phi \mid \#x \cdot \phi(x) \text{ op } k \end{aligned}$$

where  $\tau_1$  and  $\tau_2$  are terms,  $I \in \mathcal{C}$ ,  $\text{op} \in \{=, \neq, <, \leq, >, \geq\}$  and  $k \in \{0, \dots, |N|\}$ .

The formula  $\tau \in I$  asserts that the agent denoted by the term  $\tau$  is a member of the neighbourhood  $I$ .  $p @ \tau$  asserts that the condition  $p$  holds for agent  $\tau$ , and  $\gamma @ \tau$  specifies that  $\tau$  is signalling  $\gamma$  at that instant.

$\sharp$  is a counting quantifier, and  $\sharp x \cdot \phi(x) \geq k$  (for example) says that at least  $k$  agents support the assertion  $\phi$  at the instant. The modalities  $\ominus$  and  $\bigcirc$  denote the predecessor and successor instants, whereas  $\diamond$  and  $\diamond$  denote some time in the past and some time in the future, respectively. Their dual modalities are denoted  $\boxplus$  and  $\boxminus$  respectively. We talk of free and bound occurrences of variable  $x$  in formula  $\phi$  in the standard manner.  $\phi$  is said to be a sentence if it has no free occurrences of variables.

The existential and universal quantifiers are defined easily:  $\exists x \cdot \phi(x) = \sharp x \cdot \phi(x) > 0$  and  $\forall x \cdot \phi(x) = \neg \exists x \neg \phi(x)$ . We use the abbreviation  $\gamma@I = \forall x \cdot (x \in I \supset \gamma@x)$  to denote that all agents in the neighbourhood signal  $\gamma$ .

The formula  $\boxplus \forall x \cdot \gamma@x$  is denoted  $stable(\gamma)$ . The formula  $\bigvee_{\gamma \in \Gamma} stable(\gamma)$  is special and is called **emergence**.

The semantics is defined on histories. A model is a tuple  $(M, val, \eta)$ , where  $M$  is a memory system,  $\eta : V \rightarrow N$  denote an assignment of agent variables to agent names and  $val : \mathcal{Q} \rightarrow 2^{\mathcal{P}}$  is the propositional valuation map.  $val$  is lifted to configurations by the map  $\hat{val} : \mathcal{Q}^N \rightarrow (N \rightarrow 2^{\mathcal{P}})$  by:  $\hat{val}(\chi)(i) = val(\chi(i))$ .

Let  $\rho \in \mathcal{H}_M$  be a finite or infinite history  $\rho = \chi_0 \chi_1 \dots$ . The notion that  $\rho, k \models \phi$  is defined in the standard fashion, for  $k \geq 0$ .

- $\rho, k \models \tau_1 = \tau_2$  iff  $\eta(\tau_1) = \eta(\tau_2)$ .
- $\rho, k \models \tau \in I$  iff  $\eta(\tau) \in I$ .
- $\rho, k \models p@ \tau$  iff  $p \in \hat{val}(\rho_k)(\eta(\tau))$ .
- $\rho, k \models \gamma@ \tau$  iff  $\omega(\rho_k(\tau)) = \gamma$ .
- $\rho, k \models \neg \phi$  iff  $\rho, k \not\models \phi$ .
- $\rho, k \models \phi_1 \vee \phi_2$  iff  $\rho, k \models \phi_1$  or  $\rho, k \models \phi_2$ .
- $\rho, k \models \ominus \phi$  iff  $k > 0$  and  $\rho, k-1 \models \phi$ .
- $\rho, k \models \bigcirc \phi$  iff there exists a successor instant in the history and  $\rho, k+1 \models \phi$ .
- $\rho, k \models \diamond \phi$  iff there exists  $\ell \leq k$  such that  $\rho, \ell \models \phi$ .
- $\rho, k \models \diamond \phi$  iff there exists  $\ell \geq k$  such that  $\rho, \ell \models \phi$ .
- $\rho, k \models \sharp x \cdot \phi(x) \text{ op } k$  iff  $|\{j \mid \rho, k \models \phi[j/x]\}| \text{ op } k$ .

The notions of satisfiability and validity are standard. Given a model  $(M, val)$  and a sentence  $\phi$  by  $M \models \phi$  we denote that for all histories  $\rho$  in  $\mathcal{H}_M$ ,  $\rho, 0 \models \phi$ .

### 3.1 Examples

It is easily seen that specific distributions can be described in the logic. For instance consider the distribution where there are 100 agents, 30 of whom signal  $a$  and the rest signal  $b$ . We can specify this as:

$$\sharp x \cdot \gamma_a@x = 30 \wedge \sharp x \cdot \gamma_b@x = 70$$

With inequalities, classes of distributions that lead to the same signalling' behaviour can be specified.

Further the structure of interaction in histories can be constrained in the logic:

$$\exists x \cdot (x \in I \supset \boxplus(x \in I))$$



This asserts that once an agent participates in an interaction in the history, it continues to participate in every interaction in the history. (In social theory, such persistent actors in the structure are associated with memorials that keep reminding everyone of a memory token.)

The state transition structure of memory systems can be described only to a limited extent since the logic is first order and the state information which may include modular counting of signals cannot be expressed in it. However, propositional updates based on signal distributions can be specified.

The logic can describe various kinds of signalling schemes by agents.

- Signal  $a$  and  $b$  alternatively:

$$\Box[(\ominus\gamma_a@i \supset \gamma_b@i) \wedge (\ominus\gamma_b@i \supset \gamma_a@i)]$$

- If more than 5 agents in my neighbourhood previously signalled  $a$  then I signal  $a$ :

$$(i \in I \wedge \sharp x.(x \in I \wedge \ominus\gamma_a@x) > 5) \supset \gamma_a@i$$

- $\gamma$  is collective memory:

$$\Diamond\Box(\forall x.\gamma@x)$$

- $\gamma$  is collective amnesia:

$$\Diamond\Box(\forall x.\neg\gamma@x)$$

However, in terms of validities, the logic has little structure to force validities beyond that of linear time temporal logic. The interest of the logic is mainly in its role as a specification language for requirements on memory systems, and hence we are more interested in checking whether a specific memory system satisfies such a specification.

### 3.2 Model checking

In general, first order temporal logics are highly undecidable and non-axiomatizable. However, what we have here is bounded quantification, since the set of agents over which we quantify, is fixed and finite. So we can effectively eliminate quantifiers and translate formulas into propositional linear time temporal logic. Thus satisfiability is decidable via automaton construction, but yet, extracting a memory system from the formula automaton has some interesting details.

The model checking problem for the logic asks, given a model  $(M, val)$  and a sentence  $\varphi$ , whether  $M \models \varphi$ .

**Theorem 3** *The model checking problem for the logic is decidable in time linear in  $M$  and singly exponential in  $\varphi$ .*

In this case, we construct a Parikh automaton that represents the configuration space of the memory system, and take its product with the formula automaton associated with the given sentence  $\varphi$ . The construction is straightforward, though not entirely trivial.

## 4 Discussion

We began with the intention of reasoning about collective memory. How do systems of signalling in neighbourhoods embody reasoning?

Firstly, it should be clear that the history model and the logical language are rich enough to talk about the “remembrance of things past”. However, the model assumes perfect observability for agents within neighbourhoods within a fixed interaction structure. Both of these assumptions need to be relaxed, leading to epistemic logics.

Such a logical exercise is not sufficient in itself to uncover the process of *stipulation* mentioned by Sontag, or the *interdependence* between memory and reasoning demanded by Wittgenstein. However, in our opinion, the model holds considerable promise. For achieving the richness required, we hold two features to be essential: reinforcement of memory that comes through repeated interactions inside local neighbourhoods, but not confined to those neighbourhoods; complex social rules that determine influence in signalling. Elements of both are present in this model.

Further, while we have presented simple stability as the basic notion of collective memory which is persistent, we can symmetrically study notions like *collective amnesia* in the model whereby signals predominant in the system lose out in interactions and ultimately vanish. Incorporating both, to study collective memory of some events while at the same time forgetting others presents no logical difficulties but makes the model considerably complex to reason about, requiring a structural insight for simplification.

Moreover, we have taken collective memory to mean the entire system. We can instead parameterize such memory by a subset of agents to get group notions of memory; this is relevant in the study of notions such as common ground for communication ([10]).

It is interesting to consider dynamic memory systems, whereby endogenous changes in signalling behaviour can lead to altering interaction structures leading to new update rules. In particular, neighbourhoods need not be static, but may expand and contract. This is analogous to dynamic form games as we have studied elsewhere ([20]).

Social choice theory offers a variety of methods for aggregation of information from individuals for collectives, but these are static rules. Automata models such as the ones studied here can offer dynamic methods of aggregation whereby we formulate ‘local’ aggregation rules which are applied repeatedly. Whether this can lead to meaningful insight for social theories remains to be seen, offering interesting technical questions for study in the meanwhile.

Interactions in the model are nondeterministic. Stochastic models of interactions may be more appropriate for social behaviour, relevant to social network studies such as ([15], [18]). However, logicising the rationale of such interaction would perhaps need a different approach.

## 5 Acknowledgement

We thank Oliver Roy and Wang Yi for discussions on the theme, and the TARK reviewers for thoughtful remarks and helpful comments.

## References

- [1] Natasha Alechina & Brian Logan (2010): *Belief ascription under bounded resources*. *Synth.* 173(2), pp. 179–197, doi:10.1007/s11229-009-9706-6.

- [2] Thomas J Anastasio, Kristen Ann Ehrenberger, Patrick Watson & Wenyi Zhang (2012): *Individual and collective memory consolidation: Analogous processes on different levels*. MIT Press, doi:10.7551/mitpress/9173.001.0001.
- [3] Dana Angluin, James Aspnes, David Eisenstat & Erik Ruppert (2007): *The computational power of population protocols*. *Distributed Computing* 20(4), pp. 279–304, doi:10.1007/s00446-007-0040-2.
- [4] James Aspnes & Eric Ruppert (2009): *An Introduction to Population Protocols*, pp. 97–120. Springer Berlin Heidelberg, Berlin, Heidelberg, doi:10.1007/978-3-540-89707-1\_5.
- [5] Alexandru Baltag, Zoé Christoff, Rasmus K. Rendsvig & Sonja Smets (2019): *Dynamic Epistemic Logics of Diffusion and Prediction in Social Networks*. *Studia Logica* 107(3), pp. 489–531, doi:10.1007/s11225-018-9804-x.
- [6] Abhijit V. Banerjee (1992): *A Simple Model of Herd Behaviour*. *The Quarterly Journal of Economics* 107(3), pp. 797–817, doi:10.2307/2118364.
- [7] Jeffrey Andrew Barash (2016): University of Chicago Press, doi:10.7208/9780226399294-toc.
- [8] Francesco Belardinelli, Alessio Lomuscio & Emily Yu (2020): *Model Checking Temporal Epistemic Logic under Bounded Recall*. In: *Proceedings AAI 2020*, AAI Press, pp. 7071–7078, doi:10.1609/aaai.v34i05.6193.
- [9] Zoé Christoff & Jens Ulrik Hansen (2015): *A logic for diffusion in social networks*. *J. Appl. Log.* 13(1), pp. 48–77, doi:10.1016/j.jal.2014.11.011.
- [10] Herbert H Clark & Susan E Brennan (1991): *Grounding in communication*. American Psychology Association, doi:10.1037/10096-006.
- [11] C. Daskalakis & C. H. Papadimitriou (2007): *Computing equilibria in anonymous games*. In: *Proceedings of the 48th symposium on Foundations of Computer Science (FOCS)*, IEEE Computer Society Press, pp. 83–93, doi:10.1109/FOCS.2007.24.
- [12] Kaya Deuser & Pavel Naumov (2020): *On composition of bounded-recall plans*. *Artif. Intell.* 289, p. 103399, doi:10.1016/j.artint.2020.103399.
- [13] Margaret Gilbert (1987): *Modelling Collective Belief*. *Synthese* 73(1), pp. 185–204, doi:10.1007/BF00485446.
- [14] Maurice Halbwachs (1950): *The Collective Memory*. Harper and Row, New York.
- [15] David Kempe, Jon Kleinberg & Éva Tardos (2015): *Maximizing the Spread of Influence through a Social Network*. *Theory of Computing* 11(4), pp. 105–147, doi:10.4086/toc.2015.v011a004.
- [16] Felix Klaedtke & Harald Rueß (2003): *Monadic Second-Order Logics with Cardinalities*. In Jos C. M. Baeten, Jan Karel Lenstra, Joachim Parrow & Gerhard J. Woeginger, editors: *Automata, Languages and Programming*, Springer Berlin Heidelberg, pp. 681–696, doi:10.1007/3-540-45061-0\_54.
- [17] Fenrong Liu, Jeremy Seligman & Patrick Girard (2014): *Logical dynamics of belief change in the community*. *Synthese* 191(11), pp. 2403–2431, doi:10.1007/s11229-014-0432-3.
- [18] Elchanan Mossel, Joe Neeman & Omer Tamuz (2013): *Majority dynamics and aggregation of information in social networks*. *Autonomous Agents and Multi-Agent Systems* 28(3), p. 408–429, doi:10.1007/s10458-013-9230-4.
- [19] Rohit Parikh (2012): *What Is Social Software?*, pp. 3–13. LNCS 7010, Springer, doi:10.4204/EPTCS.
- [20] Soumya Paul & R. Ramanujam (2013): *Dynamics of Choice restriction in Large Games*. *IGTR* 15(4), doi:10.1142/S0219198913400318.
- [21] Soumya Paul & R. Ramanujam (2014): *Subgames within Large Games and the Heuristic of Imitation*. *Studia Logica* 102(2), pp. 361–388, doi:10.1007/s11225-014-9549-0.
- [22] William G. Roy & Rachel Parker-Gwin (1999): *How Many Logics of Collective Action? Theory and Society* 28(2), pp. 203–237, doi:10.1023/A:1006946310119.

- [23] Mihai Rusu (2017): *Transitional Politics of Memory: Political Strategies of Managing the Past in Post-communist Romania*. *Europe-Asia Studies*, pp. 260–282, doi:10.1080/09668136.2017.1380783.
- [24] Susan Sontag (2003): *Regarding the pain of others*. Picador, doi:10.3917/dio.201.0127.
- [25] John Sutton (2014): *Remembering as Public Practice: Wittgenstein, memory, and distributed cognitive ecologies*. doi:10.1515/9783110378795.409.
- [26] Allard Tamminga, Hein Duijf & Frederik Van De Putte (2020): *Expressivity results for deontic logics of collective agency*. *Synthese*, pp. 260–282, doi:10.1007/s11229-020-02597-0. Forthcoming.
- [27] Raimo Tuomela (2013): *Social Ontology: Collective Intentionality and Group Agents*. OUP USA, doi:10.1515/jso-2014-0040.
- [28] Ludwig Wittgenstein (1967): *Zettel*. Univ of California Press.